Math 155 - Trigonometric Functions

Recall Definition: Two triangles are called *similar* to each other if one can be scaled (enlarged or reduced) to get the other.

Two triangles are similar if they have the same 3 angles.

Note: If triangles share 2 angles, the 3^{rd} must be the same because they add up to 180

We will spend the rest of our time focused on triangles with a 90° angle

Definition: A triangle with a 90° angle is called a *right triangle*

Observation: If two right triangles share a second angle then, by our above Note, they must have the same third angle. Thus, the two triangles are similar.

Conclusion: All right triangles with second angle α are similar.

Example pictures of similar triangles

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Conclusion: All right triangles with second angle α are similar.

Examples pictures: 1-1.7-2, 2-3.4-4, 3-5.1-6 triangles

Notice the side opposite from the 30° angle is always half the size of the side across from the 90° (called the hypotenuse).

In other words, the ratio of $\dfrac{\textit{opposite}}{\textit{hypotenuse}} = \dfrac{1}{2}$ in all of these triangles.

Futhermore, the ratio of $\dfrac{\textit{opposite}}{\textit{hypotenuse}} = \dfrac{1}{2}$ in all of right triangles for the side opposite a 30° angle.

A similar argument can be made for any angle α, not just $\alpha = 30^\circ$

In General: For a right triangle with angle α, the ratio of $\dfrac{\textit{opposite}}{\textit{hypotenuse}}$ is the same. In particular, the amount the triangle is scaled larger or smaller does not change that ratio.

Definition: The ratio of $\dfrac{\textit{opposite}}{\textit{hypotenuse}}$ is called the $\sin(\alpha)$. i.e.

$$\sin(\alpha) = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

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Triangle example (with number for angle), find sin

Triangle example 2 (aith alpha), find sin

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Similar to the ratio of the side opposite an angle opposite to the hypotenuse, any ratio of sides stays the same in similar triangles. These ratios get define as:

Picture of triangle

$$\sin(\alpha) = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos(\alpha) = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan(\alpha) = \frac{\textit{opposite}}{\textit{adjacent}}$$

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picture of earlier triangle

Example 1: Find sin(30)

Example 2: Find cos(30)

Example 3: Find tan(30)

Example 4: Find sin(60)

Example 5: Find cos(60)

Example 6: Find tan(60)