Compound Interest cont.

We started computing interest on a loan as Simple Interest:

$$I = P \times r \times t$$

$P =$ principle, $r =$ annual interest rate, and $t =$ years of the loan
We then looked at adding interest to the loan once per year
With this, we found a formula for the loan amount A to be:

$$A = P \times (1 + r)^t$$

What if interest is added to the loan more than once per year?
Consider if we add interest to the loan - compound - 4 times per year
In each quarter of the year, simple interest is calculated with $t = \frac{1}{4}$
That is, in each quarter $I = P \times r \times \frac{1}{4} = P \times \frac{r}{4}$
Using the same logic to find A when interest was compounded yearly, we can write the amount owed after $\frac{1}{4}$ of the year as:

$$A = P + P \times \frac{r}{4} = P \times \left(1 + \frac{r}{4}\right)$$

Compound Interest cont.

The amount, A, owed on a loan with annual interest rate, r, with interest added after one quarter is:

$$A = P + P \times \frac{r}{4} = P \times \left(1 + \frac{r}{4}\right)$$

Furthermore, for each quarter year that passes, P is multiplied by another factor of $(1 + \frac{r}{4})$

For each year of the loan, interest is compounded 4 times

So, after t years, interest is compounded $4t$ times

In other words, the amount, A, owed on a loan with annual interest rate, r, compounded 4 times per year for t years is:

$$A = P \times \left(1 + \frac{r}{4}\right)^{(4 \times t)}$$

Note: P is multiplied by $\left(1 + \frac{r}{4}\right)$ each quarter, $4t$ times in t years

Compound Interest cont.

The amount, A, owed on a loan with annual interest rate, r, compounded 4 times per year for t years is:

$$A = P \times \left(1 + \frac{r}{4}\right)^{(4t)}$$

Example: How much needs to be repaid on loan of \$4000 loan with an annual interest rate $r = 3.5\%$ taken out for 6 years if the interest is compounded quarterly?

$A = P \times \left(1 + \frac{r}{4}\right)^{(4t)} = 4000 \times \left(1 + \frac{.035}{4}\right)^{(}4 \times 6)$

Computing inside the parentheses first, we find:

$A = 4000 \times (1.00875)^{24} = 4930.21$

So, the amount owed on a loan of \$4000 loan with an annual interest rate $r = 3.5\%$ taken out for 6 years, compounded quarterly is \$4930.21

Example: How much needs to be repaid on loan of \$6000 loan with an annual interest rate $r = 6\%$ taken out for 3 years if the interest is compounded quarterly?

Compound Interest cont.

What if we compound more than 4 times per year?

Most loans (car loans, credit cards, mortgages, etc.) have interest compounded monthly, or 12 times per year.

Compounding monthly, the amount of interest added each month decreases but the number of times compounding increases

We get that the amount, A, owed on a loan with annual interest rate, r, compounded 12 times per year (monthly) for t years is:

$$A = P \times \left(1 + \frac{r}{12}\right)^{12t}$$

Example: Supposed you have a car loan for \$8000 with an interest rate of 10% compounded monthly. Suppose that you can wait 1 year before making any payments, how much is owed on the loan after 1 years?

Compound Interest cont.- Classwork

In general, the amount, A, owed on a loan with annual interest rate, r, compounded n times per year for t years is:

$$A = P \times \left(1 + \frac{r}{n}\right)^{(n \times t)}$$

Example: Supposed you have a student loan for \$20000 with an interest rate of 8% compounded monthly. Suppose that you defer payment on the loan for 2 years, how much is owed on the loan after 2 years?

Example: Supposed you have a student loan for \$20000 with an interest rate of 4% compounded monthly. Suppose that you defer payment on the loan for 2 years, how much is owed on the loan after 2 years?