Math 155 - Day #7: Compound Interest

Last time we looked at computing Simple Interest on loan:

$$I = P \times r \times t$$

With simple interest, interest is added to the loan at the end depending on P = principle, r = annual interest rate, and t = years of the loan.
With most loans, interest is added to the account periodically
Example (revisited): Find the amount of interest owed on a \$2000 loan with an annual interest rate $r = 8\%$ taken out for 3 years if the interest is added to the account each year.
First, let's find out how much interest is added after 1 year.

$$I = 2000 \times .08 \times 1 = 160$$

So, after 1 year \$160 is paid in interest and
$P_1 = 2000 + 160 = 2160$ is owed on the loan after 1 year.
Now to find the interest owed after 2 years, we calculate the interest earned on the new loan amount:

$$I = 2160 \times .08 \times 1 = 172.80$$

So, the amount owed after 2 years is: $P_2 = 2160 + 172.8 = 2332.80$

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On an initial loan of $P = 2000$, we found that:

The amount owed after 1 year is: $P_1 = 2160$

The amount owed after 2 years is: $P_2 = 2332.80$

We can compute the amount owed after 3 years the same way:

$$I = 2332.80 \times .08 \times 1 = 186.62$$

$P_3 = 2332.80 + 186.62 = 2519.42$

With simple interest, the amount owed after 3 years was only 2480

The reason more is owed now is because interest was paid on the original loan and interest was paid on the interest.

This is called compound interest.

Example: How much needs to be repaid on loan of \$4000 loan with an annual interest rate $r = 3.5\%$ taken out for 6 years.

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Example: How much needs to be repaid on loan of \$4000 loan with an annual interest rate $r = 3.5\%$ taken out for 6 years.

We found that amount owed was \$4917.02

But it was tedious! How can we make it easier?

Let's look at our process to see.

Each time interest is added, how do we compute the new loan?

We compute interest: $I = P \times r \times t$ then add it back to P

That is, we compute: $P \times r \times t + P$

But notice: we can factor out P from this expression.

$$P \times (r \times t + 1)$$

So, computing the amount owed after 1 year is:
$P_1 = 4000 \times (.035 \times 1 + 1) = 4000 \times (1.035) = 4140$

The amount owed after 2 years:
$P_2 = 4140 \times (.035 \times 1 + 1) = 4140 \times (1.035) = 4284.90$

The amount owed after 3 years:
$P_3 = 4284.90 \times (.035 \times 1 + 1) = 4284.90 \times (1.035) = 4434.87$

But wait, there's more!!...

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For each time we add in interest, we multiply by $(.035 \times 1 + 1)$

After 1 year we multiply \$4000 by $(.035 \times 1 + 1)$:
$P_1 = 4000 \times (.035 \times 1 + 1)$

After 2 years we multiply \$4000 by $(.035 \times 1 + 1)$ twice:
$4000 \times (.035 \times 1 + 1) \times (.035 \times 1 + 1)$

But multiplying by something twice is:
$(.035 \times 1 + 1) \times (.035 \times 1 + 1) = (.035 \times 1 + 1)^2$

$P_2 = 4000 \times (.035 \times 1 + 1)^2$

After 3 years we multiply \$4000 by $(.035 \times 1 + 1)$ three times:
$4000 \times (.035 \times 1 + 1)^3$

After 6 years we multiply \$4000 by $(.035 \times 1 + 1)$ six times:
$4000 \times (.035 \times 1 + 1)^6 = 4917.02$

Conclusion: If we have a $\$P$ loan with an annual interest rate r compounded each year, then the amount owed A after t years is:

$$A = P \times (1 + r)^t$$

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Example: How much is owed on a loan of 8000 with an interest rate of 6% taken out for 10 years if the interest is compounded each year?

Example: How much is owed on a loan of 5000 with an interest rate of 8% taken out for 7 years if the interest is compounded each year?