Math 155 - Day #23: Probability - Independent Events

Recall: our definition of the probability of an event A occurring is:

$$P(A) = \frac{\text{\# of ways event } A \text{ can occur}}{\text{\# of possible outcomes}}$$

And we found some properties of probabilities:

1. For any event A, $0 \leq P(A) \leq 1$
2. For any event A, $P(\text{not } A) = 1 - P(A)$
3. If events A and B cannot happen at the same time, then $P(A \text{ or } B) = P(A) + P(B)$

There is one more important property for probabilities when two events are independent - meaning the outcome of one does not effect the outcome of the other.

If A and B are independent events then:

$$P(A \text{ and } B) = P(A) \times P(B)$$

Example: What is the probability of flipping a coin and getting heads twice in a row?

$$P(\text{heads and heads}) = P(\text{heads on first flip}) \times P(\text{heads on second flip})$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

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If A and B are independent events then:

$$P(A \text{ and } B) = P(A) \times P(B)$$

To see why this is true, we start on the right side of the equation:

$$P(A) \times P(B) = \frac{\#\text{ ways } A \text{ can occur}}{\#\text{ outcomes of } 1^{st} \text{ event}} \times \frac{\#\text{ ways } B \text{ can occur}}{\#\text{ outcomes of } 2^{nd} \text{ event}}$$

Multiplying this fraction across top and bottom we have:

$$P(A) \times P(B) = \frac{\left(\#\text{ ways } A \text{ can occur}\right) \times \left(\#\text{ ways } B \text{ can occur}\right)}{\left(\#\text{ outcomes of } 1^{st} \text{ event}\right) \times \left(\#\text{ outcomes of } 2^{nd} \text{ event}\right)}$$

In both the top and bottom, we can use the Fundamental Counting Principle:

$$P(A) \times P(B) = \frac{\#\text{ ways } A \text{ and } B \text{ can occur}}{\#\text{ outcomes of } 1^{st} \text{ and } 2^{nd} \text{ events}} = P(A \text{ and } B)$$

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Example: You roll 3 dice. What is the probability of getting a 5 on all three?

Since the outcome of each die is independent of the others:

$P(5 \text{ and } 5 \text{ and } 5) = P(5) \times P(5) \times P(5) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

Example: You roll 3 dice. What is the probability of getting a no 5's?

$P(\text{not } 5 \text{ and not } 5 \text{ and not } 5) = P(\text{not } 5) \times P(\text{not } 5) \times P(\text{not } 5) = \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{6}\right) \times \left(1 - \frac{1}{6}\right) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

Example: You roll 3 dice. What is the probability of getting at least one 5?

$P(\text{at least one } 5) = P(\text{not zero } 5\text{'s}) = 1 - P(\text{no } 5\text{'s}) = 1 - \frac{125}{216} = \frac{91}{216}$

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Example: Suppose you roll 2 dice. What is the probability of getting two 5's?

$P(5 \text{ and } 5) = P(5) \times P(5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Example: Suppose you roll 2 dice. What is the probability of getting no 5's?

$P(\text{no 5's}) = P(\text{not } 5) \times P(\text{not } 5) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

Example: Suppose you roll 2 dice. What is the probability of getting one 5?

$P(\text{one } 5) = 1 - P(\text{not one } 5) = 1 - P(\text{no 5's or one } 5) =$
$1 - [P(\text{no 5's}) + P(\text{one } 5)] = 1 - \left[\frac{25}{36} + \frac{1}{36}\right] = 1 - \frac{25}{36} = \frac{10}{36}$

Note: Since, these are the only three possible outcomes, the sum of the probabilities should be 1. Is it?