Math 155 - Counting & Probability Review

Our methods of counting all rely on one core idea:

Fundamental Counting Principle: If you have a choices from one set and b choices from another set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\#\text{ of choices from set 1}} \times \underbrace{b}_{\#\text{ of choices from set 2}}$$

Example: Suppose that your professor has 5 shirts, 3 pairs of pants, and a willingness to wear them in any combination. How many different outfits can he wear?

The total number of outfits is $\underbrace{5}_{shirts} \times \underbrace{3}_{pants} = 15$

Note: this can be extended to more than 2 choices.

This allowed us to find the number of ways n objects can be ordered:

The # of ways n objects can be ordered is: $n! = n \times (n-1) \times \cdots \times 2 \times 1$

Example: 4 objects can be ordered in: $4! = 4 \times 3 \times 2 \times 1 = 24$ ways

Math 155 - Counting & Probability Review

Sometimes we only want to order some objects in a set.

Permutations: The number of ways to order k things from a set of n is called a permutation and is written as:

$$_nP_k = \frac{n!}{(n-k)!} = n \times (n-1) \times \cdots \times (n-k+1)$$

Example: How many ways can you line up 5 people from a group of 10?

Answer: $_{10}P_5 = \frac{10!}{5!} = 30240$

If the order does not matter, and we only want to pick k items from n then this is called a Combination, and differs from a Permutation by a factor of k!

Combinations: The number of ways to choose k things from a set of n is called a combination and is written as:

$$_nC_k = \frac{n!}{(n-k)!k!}$$

Example: How many 5 card hands are there in a 52 card deck?

Card order doesn't matter, so we use a combination: $_{52}C_5 = 2598960$

Math 155 - Counting & Probability Review

The probability of an event is the likelihood that the event occurs.
We write the probability of event A as: $P(A)$

$$P(A) = \frac{\#\text{ of ways event } A \text{ can occur}}{\#\text{ of possible outcomes}}$$

Example: Rolling a 6-sided die, what is the probability of an even roll?
There are 6 possible outcomes, with 3 outcomes that are even: 2, 4, 6
So, $P(\text{rolling an even}) = \frac{3}{6} = \frac{1}{2}$
This turns probability problems into 2 counting problems. Because counting problems vary greatly in difficulty, so do probabilities.

Rules of Probabilities

1. For any event A, $0 \leq P(A) \leq 1$
2. For any event A, $P(\text{not } A) = 1 - P(A)$
3. If events A and B cannot happen at the same time, then $P(A \text{ or } B) = P(A) + P(B)$
4. If A and B are independent events then:

$$P(A \text{ and } B) = P(A) \times P(B)$$