Math 155 - Day #22: Rules for Probability

We write the probability of event A as: $P(A)$

$$P(A) = \frac{\# \text{ of ways event } A \text{ can occur}}{\# \text{ of possible outcomes}}$$

There are several rules that dictate what is a probability

Rules of Probabilities

1. For any event A, $0 \leq P(A) \leq 1$

Since $1 = 100\%$ This says that the probability of an event occurring must be between 0% and 100%

2. For any event A, $P(\text{not } A) = 1 - P(A)$

Note: We sometimes write "not A" as A^C and call it "A complement"

Example: If the probability that event A occurs is $P(A) = .2 = 20\%$ then the probability that A does not occur is
$P(\text{not } A) = 1 - .2 = .8 = 80\%$

3. If events A and B cannot happen at the same time, then
$P(A \text{ or } B) = P(A) + P(B)$

Math 155 - Day #22: Rules for Probability

To explain the third property, let's go back to the probability definition

If A and B cannot happen at the same time, then:

$$
\begin{aligned}
P(A \text{ or } B) &= \frac{\text{\# of ways } A \text{ or } B \text{ can occur}}{\text{\# of possible outcomes}} \\
&= *\frac{\text{\# of ways } A \text{ can occur} + \text{\# of ways } B \text{ can occur}}{\text{\# of possible outcomes}} \\
&= \frac{\text{\# of ways } A \text{ can occur}}{\text{\# of possible outcomes}} + \frac{\text{\# of ways } B \text{ can occur}}{\text{\# of possible outcomes}} \\
&= P(A) + P(B)
\end{aligned}
$$

*Note: These are only equal if A and B cannot happen at the same time. Otherwise, events that happen in both A and B get counted twice.

To work with these properties, we will build an example first

We will find the probabilities of dice rolls when rolling two dice and adding them together

Math 155 - Day #22: Rules for Probability

Suppose that you roll two fair dice then add their results together. The table below shows the outcome of each die with what their sum is.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Math 155 - Day #22: Rules for Probability

Example: What is the probability of rolling an 8?

There are 5 outcomes where the dice sum to 8 out of 36 possible dice rolls

So, $P(\text{rolling an 8}) = \frac{5}{36}$

Example: What is the probability of rolling a 10?

There are 3 outcomes where the dice sum to 10 out of 36 possible dice rolls

So, $P(\text{rolling a 10}) = \frac{3}{36}$

Example: What is the probability of not rolling a 7?

$P(\text{not } 7) = 1 - P(7) = 1 - \frac{6}{36} = \frac{30}{36}$

Example: What is the probability of rolling a 5 or 6?

$P(5 \text{ or } 6) = P(5) + P(6) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36}$

Example: What is the probability of rolling less than 10?

$P(\text{less than } 10) = 1 - P(\text{greater than or equal to } 10) =$
$1 - [P(10) + P(11) + P(12)] = 1 - \left[\frac{3}{36} + \frac{2}{36} + \frac{1}{36}\right] = 1 - \frac{6}{36} = \frac{30}{36}$