Math 155 - Day #16: Fundamental Counting Principle

We have all learned to count sequentially: 1, 2, 3, 4, etc.

If we have a large quantity that we need to count, we need to count strategically so that we include everything and miss nothing!

In the upcoming classes, we will learn different methods of counting used in different situations.

All of the counting methods we will learn have a common principle behind them called: The Fundamental Counting Principle

Before stating it formally, let's do some warm-up examples!

Example: Suppose that your professor has 3 shirts, 2 pairs of pants, and a willingness to wear them in any combination. How many different outfits can he wear?

This is a counting problem: we are counting the number of outfits

Shirt 1, Pants 1

Shirt 1, Pants 2

Shirt 2, Pants 1

Shirt 2, Pants 2

Shirt 3, Pants 1

Shirt 3, Pants 2

There are 6 total outfits

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Example: Suppose that your professor has 4 shirts, 2 pairs of pants, and a willingness to wear them in any combination. How many different outfits can he wear?

Shirt 1, Pants 1

Shirt 1, Pants 2

Shirt 2, Pants 1

Shirt 2, Pants 2

Shirt 3, Pants 1

Shirt 3, Pants 2

Shirt 4, Pants 1

Shirt 4, Pants 2

There are 8 total outfits

Notice that counting in this ways groups all combinations with Shirt 1 together, Shirt 2 together, and so on.

For each shirt we pick there are 2 choices of pants.

And we have 4 choices for which shirt to pick.

Using this thinking, we can see that we have $\underbrace{4}_{shirts} \times \underbrace{2}_{pants} = 8$ choices.

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Fundamental Counting Principle: If you have a choices from one set and b choices from another set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\#\text{ of choices from set 1}} \times \underbrace{b}_{\#\text{ of choices from set 2}}$$

Example: Suppose that your professor has 5 shirts, 3 pairs of pants, and a willingness to wear them in any combination. How many different outfits can he wear?

The total number of outfits is $\underbrace{5}_{shirts} \times \underbrace{3}_{pants} = 15$

Using this method is easier than counting each combination one at a time, especially if the numbers get large.

Example: Suppose that your professor has 50 shirts, 30 pairs of pants, and a willingness to wear them in any combination. How many different outfits can he wear?

The total number of outfits is $\underbrace{50}_{shirts} \times \underbrace{30}_{pants} = 1500$

Note: I would NOT want to make a list that long to count all the combinations!

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Fundamental Counting Principle: If you have *a* choices from one set and *b* choices from another set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\#\text{ of choices from set 1}} \times \underbrace{b}_{\#\text{ of choices from set 2}}$$

But what if we have more than 2 things to choose from?

Example: Suppose that your professor has 5 shirts, 3 pairs of pants, and 2 pairs of shoes. How many different outfits can he wear?

How do we count these without going back to the old way of writing out each combination and adding them up?

One way to think about it is that we *are* making 2 choices, not 3. The first choice we make is shirt/pant combo (the 15 options we found before) and the second choice is which pair of shoes (2 choices)

So, the total number of choices is: $15 \times 2 = 30$

Remember, 15 is from: $5 \times 3 = 15$ choices for pants/shirt combos

More Generel Using this thinking, we find that if you have *a* choices from one set and *b* choices from another set and *c* choices from third set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\#\text{ of choices from set 1}} \times \underbrace{b}_{\#\text{ of choices from set 2}} \times \underbrace{c}_{\#\text{ of choices from set 3}}$$

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General Fundamental Counting Principle: If you have n sets of choices to make with a_1 choices from one set, a_2 choices from the next set, ... , a_n choices from the last set, then the number of ways that you can pick one item from each is:

$$\underbrace{a_1}_{\#\text{ of choices from set 1}} \times \underbrace{a_2}_{\#\text{ of choices from set 2}} \times \cdots \times \underbrace{a_n}_{\#\text{ of choices from}}$$

Example: Suppose that your professor has 5 shirts, 3 pairs of pants, 2 pairs of shoes, and 7 ties. How many different outfits can he wear?
The number of outfits $= 5 \times 3 \times 2 \times 7 = 210$

Example Suppose that you have 12 shirts, 9 pairs of pants, and 3 hats. How many combinations of outfits can you make?

$$\text{Total outfits} = \underbrace{12}_{\text{shirts}} \times \underbrace{9}_{\text{pants}} \times \underbrace{3}_{\text{hats}} = 324$$

Example Suppose that you have one red dice and one blue dice (both standard 6-sided die). How many different combinations of dice rolls can you get?

$$\text{Total rolls} = \underbrace{6}_{\text{red die}} \times \underbrace{6}_{\text{blue die}} = 36$$