Math 155 - Day #18: Examples - FCP & Permutations

Fundamental Counting Principle: If you have *a* choices from one set and *b* choices from another set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\text{\# of choices from set 1}} \times \underbrace{b}_{\text{\# of choices from set 2}}$$

Example: Typical Massachusetts license plates are made up of 2 numbers, followed up 2 letters, then two more numbers. How many possible license plates can be made with this pattern?

Answer: Using the FCP and that we have 10 choices for the 1st, 2nd, 5th, and 6th spots with 26 choices in the 3rd and 4th spots, we have:
Total choices $= 10 \times 10 \times 26 \times 26 \times 10 \times 10 = 6760000$

Example: Suppose that a password consists of 8 lowercase letters. How many possible passwords can be made?

Answer: Here there are 8 characters, each with 26 choices
Total choices $= 26^8 = 208827064576 \approx 208 \, billion$

Example: Stronger passwords include Uppercase letters, numbers, and symbols such as \$&% with a total of 92 possible choices for each character. How many possible 8-character passwords can be made?

Answer: Here there are 8 characters, each with 92 choices
Total choices $= = 92^8 = \approx 5 \, quadrillion$

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Example: Suppose that you have 9 baseball players to put into a batting order. How many ways can the 9 players be ordered?

Answer: $9! = 362880$

Example: Suppose that you have a baseball team with 15 position players (non-pitchers). How many ways can you create a 9-person batting order from those 15 people?

Answer: $_{15}P_9 = \dfrac{15!}{(15-9!)} = \dfrac{15!}{(6!)} = 1816214400$

Example: Suppose that you want to put 8 books on a bookshelf - the first 5 will be Math books and the next 3 will be Psychology books. You have 12 Math books and 8 Psychology books to choose from. How many different orderings are there if you put the 5 Math books on the shelf first?

Answer: There are $_{12}P_5 = 95040$ ways of ordering the Math books and $_8P_3 = 336$ ways of ordering the Psychology books. Since we have 95040 choices for the first set of books and 336 choices for the second set of book, we have:

Total choices $= 95040 \times 336 = 31933440 \approx 32 million$