Math 155 - Day #17: Permutations

Last time we found Fundamental Counting Principle:

Fundamental Counting Principle: If you have a choices from one set and b choices from another set, then the number of ways that you can pick one item from each is:

$$\underbrace{a}_{\#\text{ of choices from set 1}} \times \underbrace{b}_{\#\text{ of choices from set 2}}$$

General Fundamental Counting Principle: If you have n sets of choices to make with a_1 choices from one set, a_2 choices from the next set, ... , a_n choices from the last set, then the number of ways that you can pick one item from each is:

$$\underbrace{a_1}_{\#\text{ choices set 1}} \times \underbrace{a_2}_{\#\text{ choices set 2}} \times \cdots \times \underbrace{a_n}_{\#\text{ choices set } n}$$

These results will be our guide for many of the counting principles that we learn

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$$\underbrace{a_1}_{\#\text{ choices set 1}} \times \underbrace{a_2}_{\#\text{ choices set 2}} \times \cdots \times \underbrace{a_n}_{\#\text{ choices set } n}$$

Example: Suppose that you have 4 people to line up. How many possible ways can this be done?

Here we have 4 choices: Who is first, second, third, and fourth

We will use the FCP to find how many possible ways there are.

$$\text{Total }^{\#}\text{ of ways} = \underbrace{\underline{\qquad\qquad}}_{1^{st}\text{ spot}} \times \underbrace{\underline{\qquad\qquad}}_{2^{nd}\text{ spot}} \times \underbrace{\underline{\qquad\qquad}}_{3^{rd}\text{ spot}} \times \underbrace{\underline{\qquad\qquad}}_{4^{th}\text{ spot}}$$

We have 4 people to choose from for the first spot

How many choices for the 2^{nd} spot?

Since one person in first spot, only 3 choices left for second spot

Similarly, there are 2 choices for 3^{rd} and 1 choice for 4^{th} spots

Total choices $= 4 \times 3 \times 2 \times 1 = 24$

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Example: Suppose that you have 6 people to line up. How many possible ways can this be done?

Total $^{\#}$ of choices $= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Notation: $n! = n \times (n-1) \times \cdots \times 3 \times 2 \times 1$

This is called a factorial.

Re-writing the above example, we can say:

Total $^{\#}$ of choices $= 6!$

Example: Suppose that you have 8 people to line up. How many possible ways can this be done?

Total $^{\#}$ of choices $= 8! = 40320$

Notice that these numbers grow very quickly!

Example: Suppose that you have 20 people to line up. How many possible ways can this be done?

Total $^{\#}$ of choices $= 20! \approx 2.4 quintillion$

Example: Suppose that you have 0 people to line up. How many possible ways can this be done?

1: the one and only choice is to not line anyone up

Note: We define $0! = 1$

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Suppose that you have 8 people and want to line up 4 of them.

Total # of ways $= \underbrace{\_\_\_\_\_\_}_{1^{st}\text{ spot}} \times \underbrace{\_\_\_\_\_\_}_{2^{nd}\text{ spot}} \times \underbrace{\_\_\_\_\_\_}_{3^{rd}\text{ spot}} \times \underbrace{\_\_\_\_\_\_}_{4^{th}\text{ spot}}$

We have 8 people to choose from for the first spot

How many choices for the 2^{nd} spot?

Since one person in first spot, only 7 choices left for second spot

Similarly, there are 6 choices for 3^{rd} and 5 choice for 4^{th} spots

Total choices $= 8 \times 7 \times 6 \times 5 = 1680$

Can this be written with factorials like before?

It starts out similar to 8! but stops at 5

In other words, it starts as 8! but is missing $4 \times 3 \times 2 \times 1 = 4!$

Combining this, we can write it as:

$$8 \times 7 \times 6 \times 5 = \frac{8 \times 7 \times 6 \times 5 \cancel{\times 4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1}} = \frac{8!}{4!}$$

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In General: The number of ways to order k things from a set of n is called a permutation and is written as:

$$ {}_nP_k = \frac{n!}{(n-k)!} = n \times (n-1) \times \cdots \times (n-k+1) $$

Example: How many ways can you line up 5 people from a group of 10?

Answer: ${}_{10}P_5 = \frac{10!}{5!} = 30240$

Example: How many ways can you line up 5 people from a group of 5?

Example: How many ways can you line up 5 people from 12?

Example: How many ways can you line up 3 people from 100?