# Math 155 - Day #19: Combinations

**Permutations:** The number of ways to order $k$ things from a set of $n$ is called a permutation and is written as:

$$_nP_k = \frac{n!}{(n-k)!}$$

What if we only want to choose part of a group, but not order them?

**Example:** Suppose that a town has 5 candidates for City Council and voters can choose 3 of them. How many ways can a voter choose 3 candidates from the set of 5?

If the order does not matter then this is call a *Combination*.

We can write the combinations as $_5C_3$

Let's try to count the number of possibilities by labeling the candidates A, B, C, D, E

ABC ABD ABE ACD

Note: There's no ACB since it's the same as ABC as a combination

ACE ADE BCD BCE BDE CDE - There are 10 total

So, $_5C_3 = 10$

As we've seen with Permutations, Combinations can get very large

So, we need to find a more strategic way to count them!

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What is the different between a combination and permutation?
A Combination $({}_nC_k)$ chooses $k$ items from a set of $n$.
A Permutation $({}_nP_k)$ chooses $k$ items from a set of $n$ and orders $k$ items
Using the ideas from the Fundamental Counting Principle, since Permutations come from two choices (which $k$ items from a set of $n$ and which order for the $k$ items) we have that:

$$_nP_k = \left(^{\#} \text{ ways to choose } k \text{ from } n\right) \times \left(^{\#} \text{ of ways to order } k \text{ items }\right)$$

The two things on right are the combinations we're trying to understand and the ordering of $k$ items, which we've already found to be $k$!
So, we can re-write this equation as: $_nP_k =_n C_k \times k!$
So, we can solve this for $_nC_k$ by dividing by $k!$ to get: $_nC_k = \frac{_nP_k}{k!}$
Substituting $_nP_k = \frac{n!}{(n-k)!}$ in this equation, we are left with our formula:

$$_nC_k = \frac{n!}{(n-k)! \times k!}$$

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**Example:** Suppose that you have 15 position players on a baseball team and need to choose 9 of them to be the starting players in a game. Notice here, we are not talking batting order - or anything that orders them. Just if they are in the starting group or not.
So, we need to use a combination of 9 from 15
$_{15}C_9 = 5005$
**Observation:** Notice that we could have equivalently chosen which 6 players did not start
$_{15}C_6 = \frac{15!}{9!6!} = {}_{15}C_9$
There is symmetry in combinations that $_nC_k = {}_nC_{(n-k)}$
**Example:** How many combinations of 2 Kings are there in a standard deck of 52 cards with 4 Kings?
In this example, we are choosing 2 Kings from a set of 4.
$_4C_2 = 6$

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**Example:** How many ways can you be dealt 2 Kings in a 5-card hand from a standard 52-card deck?

Here, we have two things to consider: How many ways can the 2 Kings be chosen from the 4 possible Kings AND how many ways can the other 3 cards be chosen from the 48 non-King cards

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**Example:** How many ways can you choose 5 crayons from a set of 16 to put on a table for a child to draw with?

**Example:** How many ways can a 5-card hand be dealt from a 52-card deck?

**Example:** How many ways can you be dealt a 5-card hand with 3 Diamonds?

Note: There are 13 cards of each suit (including Diamonds) in a standard deck of cards