

## Graphing Rational Functions - Example 7

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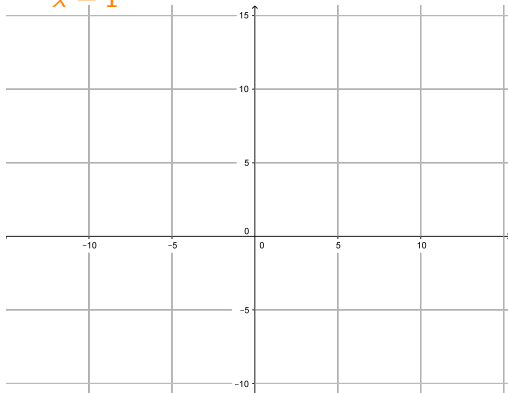
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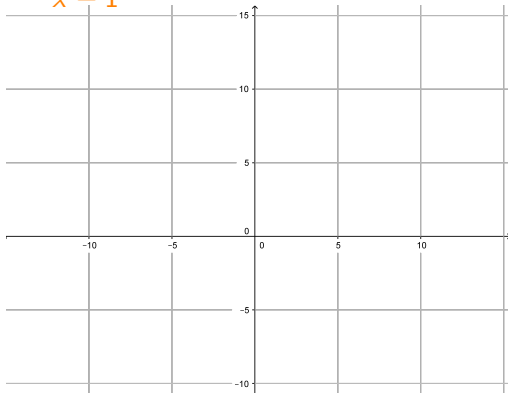
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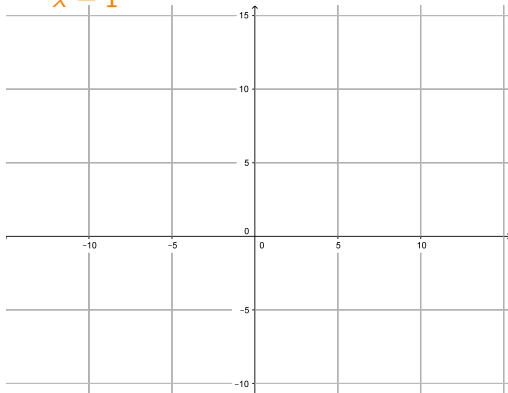
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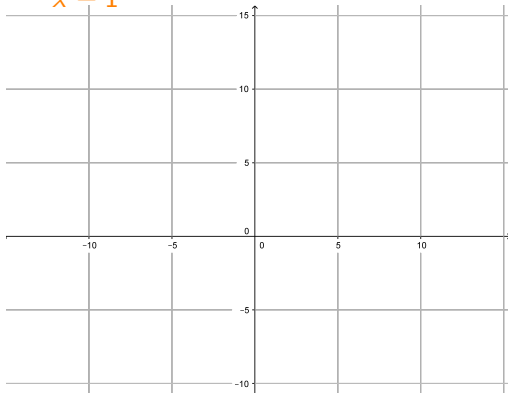
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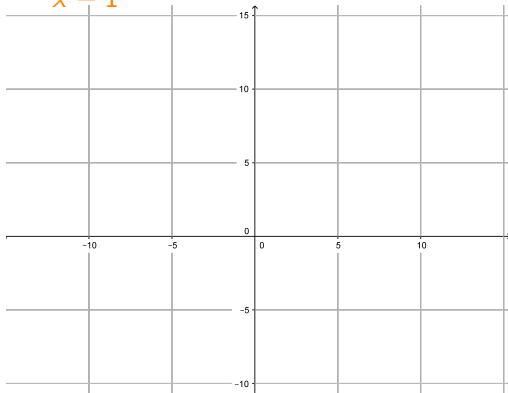
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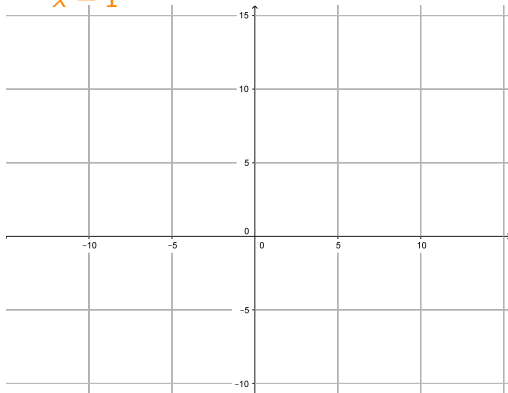
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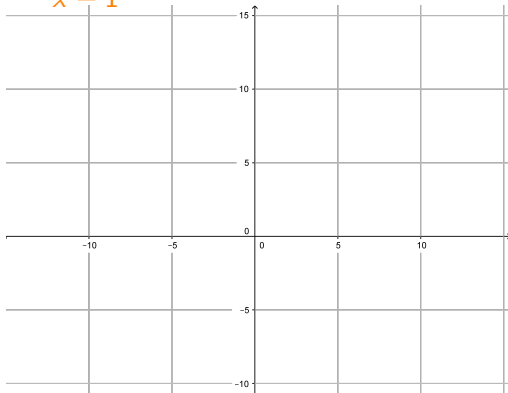
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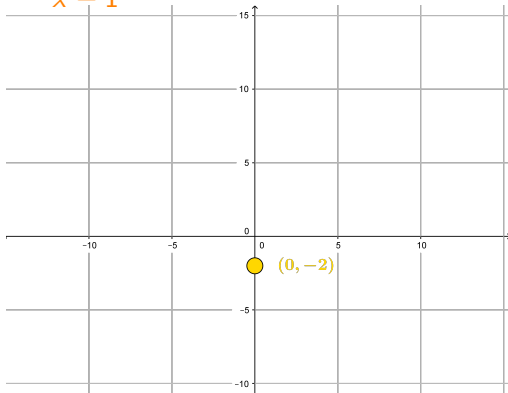
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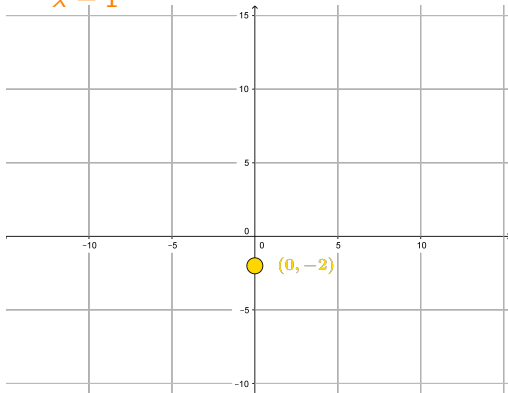
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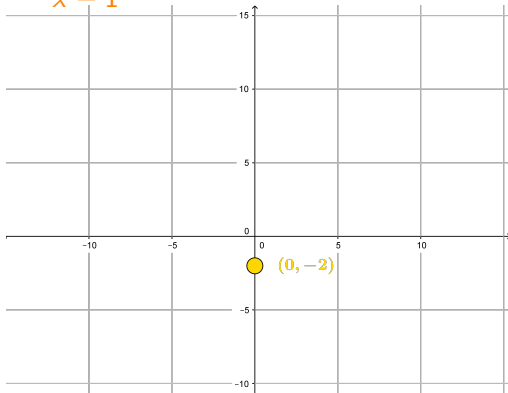
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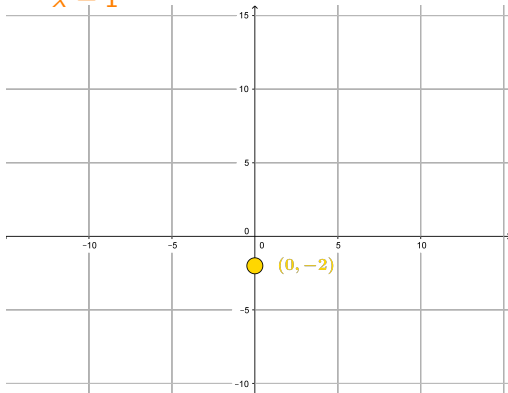
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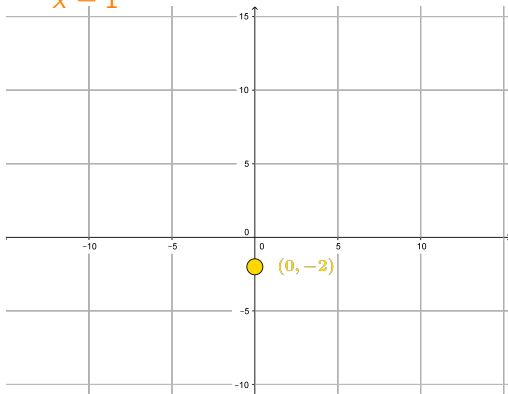
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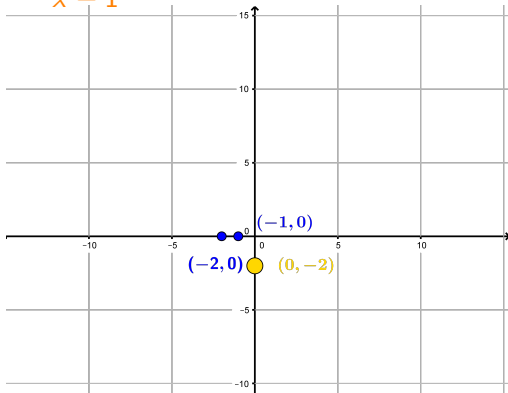
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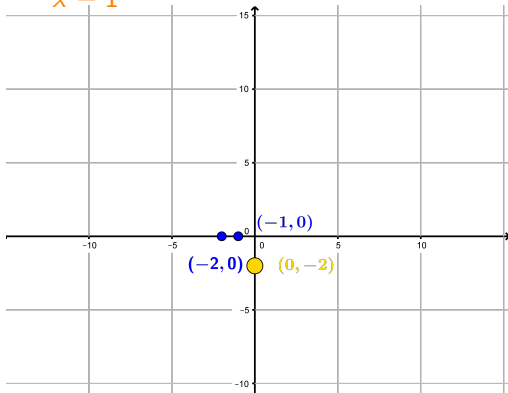
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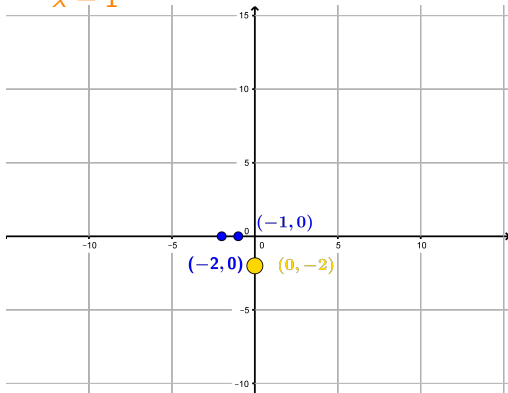
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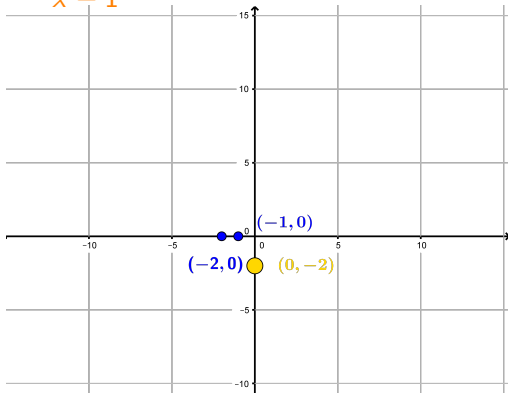
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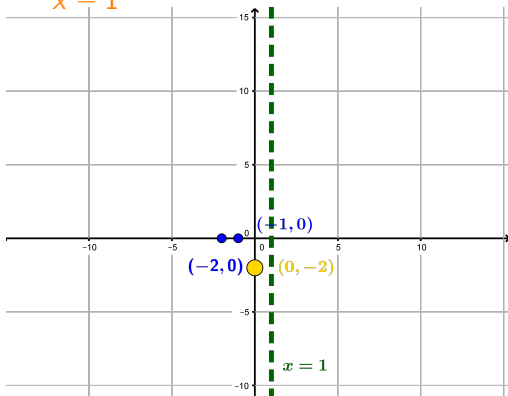
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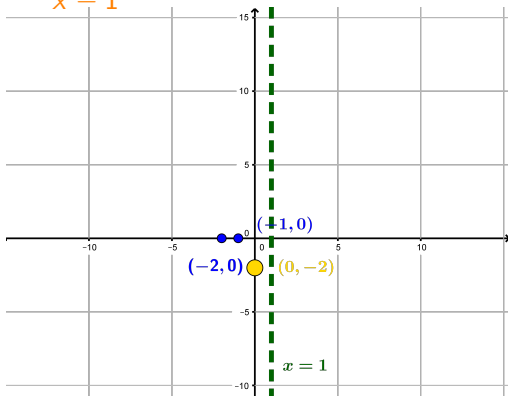
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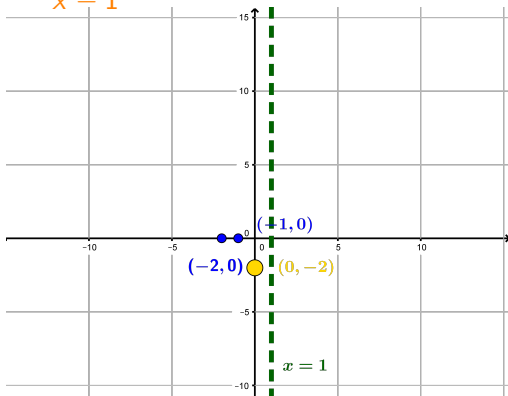
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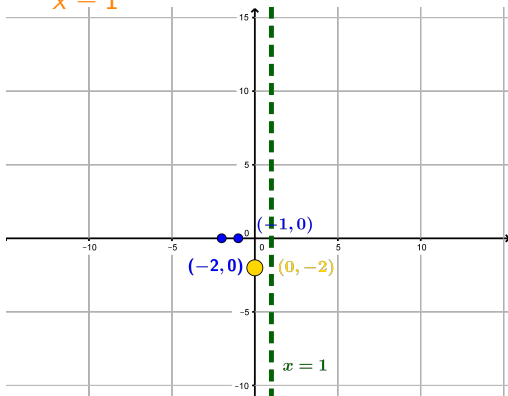
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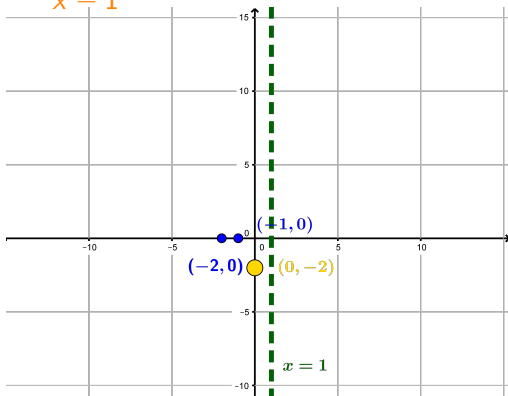
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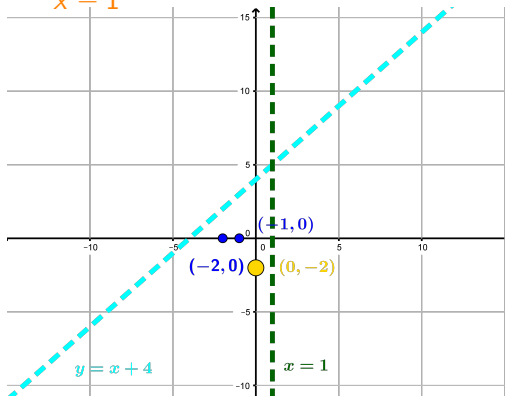
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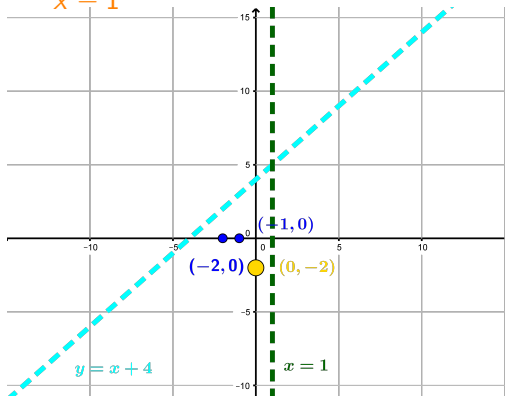
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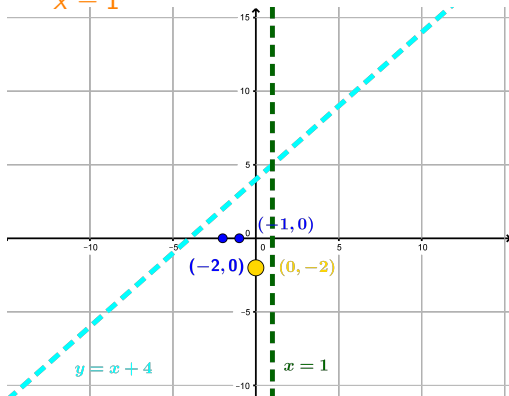
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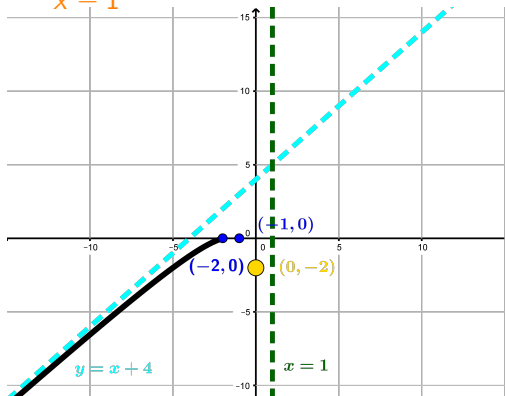
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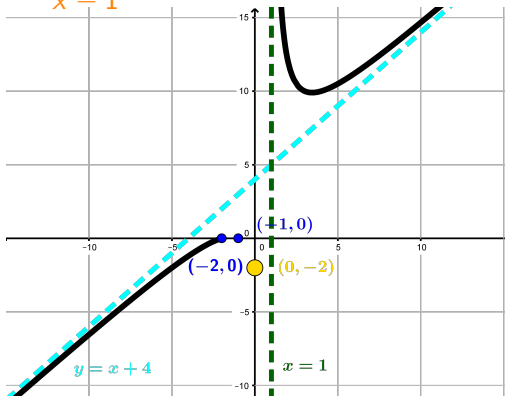
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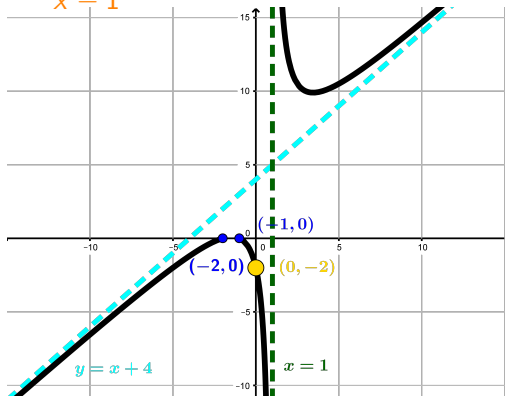
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► We saw that since  $\deg(x^2 + 3 \cdot x + 2) > \deg(x - 1)$  we need to ► Divide

$$\frac{x^2 + 3 \cdot x + 2}{x - 1} = x + 4 + \frac{6}{x - 1} \text{ so } f(x) \approx x + 4 \text{ for } x \rightarrow \pm\infty$$

► Like Polynomials we need to check if  $f(x) > 0$  or  $f(x) < 0$  on some intervals

Since there are no more  $x$ -int we know where  $f(x)$  can change sign



## Graphing Rational Functions - Example 7

**Example:** Sketch the graph of:

$$f(x) = \frac{x^2 + 3 \cdot x + 2}{x - 1}$$

We need to find:

The  $y$ -int:  $x = 0$

$$f(0) = \frac{0^2 + 3 \cdot 0 + 2}{0 - 1} = -2 : (0, -2)$$

The  $x$ -int:  $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{x^2 + 3 \cdot x + 2}{x - 1}$$

► We can by solving:

$$0 = x^2 + 3 \cdot x + 2$$

This gives:  $x = -1, -2$

Vertical asymptotes:  $x - 1 = 0$

Solving this gives:  $x = 1$

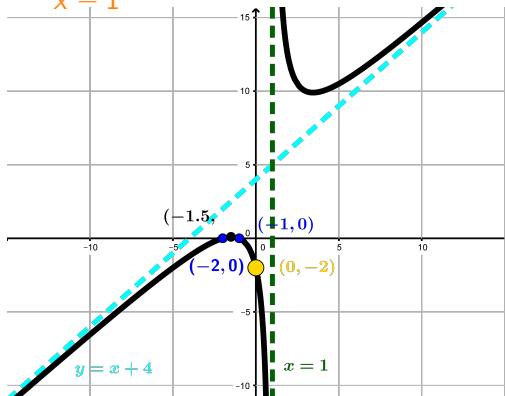
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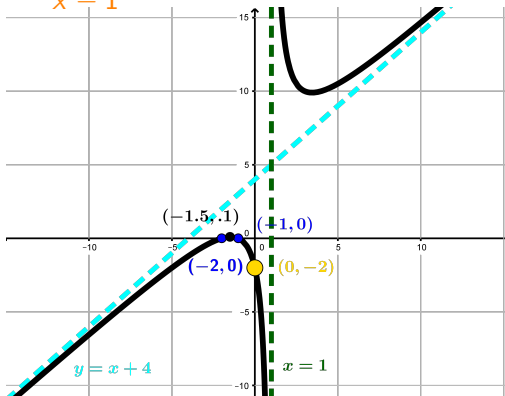
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