

Solving Quadratics without Factoring - Example

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$$(x + 2)^2 - 3 = 0$$

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To solve for x we have to undo 3 operations, which are:

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To solve for x we have to undo 3 operations, which are:

Add 2

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$$(x+2)^2 - 3 = 0$$

To solve for x we have to undo 3 operations, which are:

Add 2

Square it

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Example: Find the solution(s) to:

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To solve for x we have to undo 3 operations, which are:

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Subtract 3

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▶ Like Frog and Toad

we undo these operations in the opposite order.

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Now, we can Take the Square Root of both sides to get:

$$\sqrt{(x+2)^2} = \sqrt{3}$$

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$$|x+2| = \sqrt{(x+2)^2} = \sqrt{3} = \sqrt{3}$$

Since $|x+2| = \sqrt{3}$, we know that $x+2 = \pm\sqrt{3}$

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Subtracting 2 from both sides gives the solutions: $x = -2 \pm \sqrt{3}$

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Since $|x+2| = \sqrt{3}$, we know that $x+2 = \pm\sqrt{3}$

Subtracting 2 from both sides gives the solutions: $x = -2 \pm \sqrt{3}$

We can write the solutions as a list:

$$x = -2 + \sqrt{3}, -2 - \sqrt{3}$$

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$$(x + 2) \cdot (x + 2) - 3 = 0$$

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$$(x + 2)^2 - 3 = 0$$

Let's take a second look at our example.

What if we had multiplied out the left hand side?

$$x^2 = (x + 2) \cdot (x + 2) - 3 = 0$$

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What if we had multiplied out the left hand side?

$$x^2 + \underbrace{2x + 2x}_{4x} + \underbrace{4 - 3}_1 = (x + 2) \cdot (x + 2) - 3 = 0$$

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Leaving us with the equation:

$$x^2 + 4x + 1 = 0$$

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$$x^2 + 4x + 1 = (x + 2)^2 - 3$$

x only shows up in one place if we write the quadratic as: $(x + 2)^2 - 3$

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This question becomes:

If we are given an equation in the form: $ax^2 + bx + c = 0$

How do we write it in the form: $a(x - h)^2 + k$

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Because once we write the quadratic in this form, we can solve it!

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We will see soon why having $-h$ and $+k$, is useful