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A consequence of The Fundamental Theorem of Algebra is that every polynomial of degree n has n roots.

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Question: Why does having at least 1 root mean that there are n roots?

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$$P(k) = 0 \qquad \iff \qquad P(x) = (x - k) \cdot Q(x)$$

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This continues n times until we get to $deg = 0$, which is a constant!

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