

Pythagorean Theorem

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

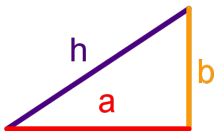
Proof:

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



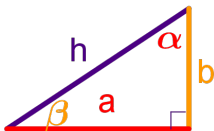
Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:

We label the angles α and β



Pythagorean Theorem

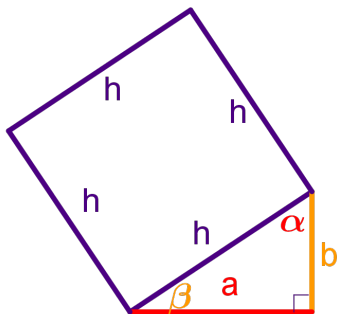
Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:

We label the angles α and β

We bring h^2 into the picture



Pythagorean Theorem

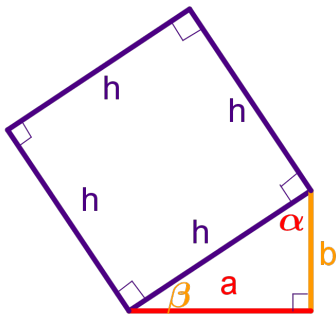
Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:

We label the angles α and β

We bring h^2 into the picture

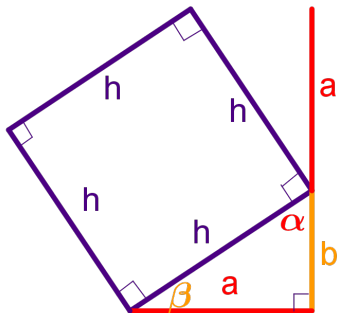


Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



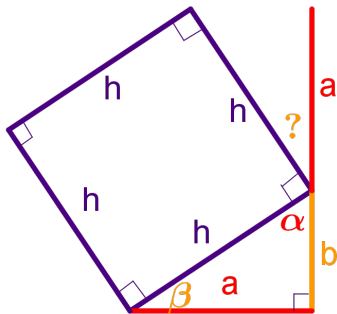
We label the angles α and β
We bring h^2 into the picture
To begin our next square, we
extend b by length a

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



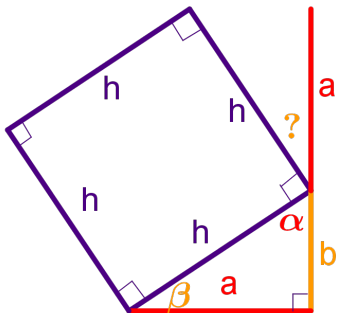
We label the angles α and β
We bring h^2 into the picture
To begin our next square, we
extend b by length a

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we
extend b by length a

To find this angle, we use
that $90 + \alpha + \beta = 180$

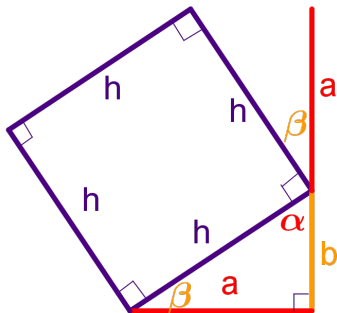
And $90 + \alpha + ? = 180$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we
extend b by length a

To find this angle, we use
that $90 + \alpha + \beta = 180$

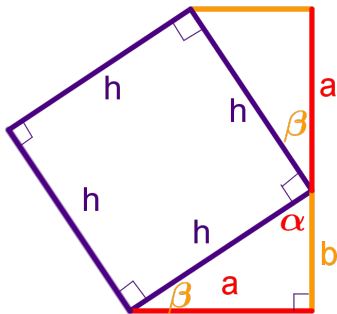
And $90 + \alpha + ? = 180$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we
extend b by length a

To find this angle, we use
that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

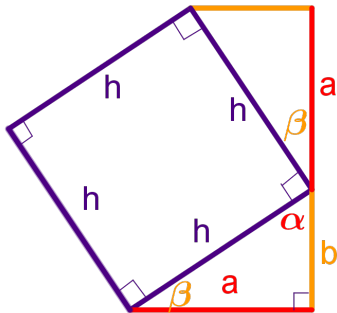
We complete this triangle

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

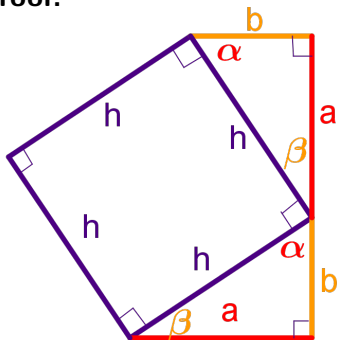
The triangles share h, β, a

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

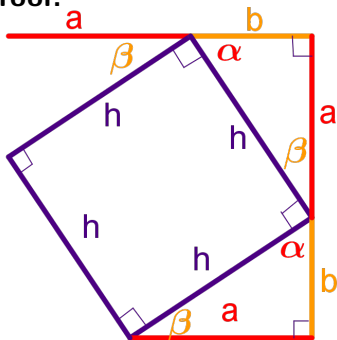
The triangles are congruent

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

The triangles are congruent

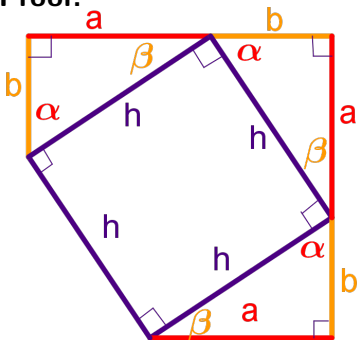
Continuing this process

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

The triangles are congruent

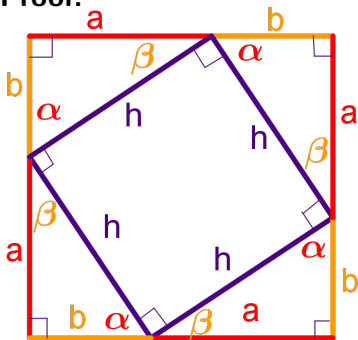
Continuing this process

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

The triangles are congruent

Continuing this process

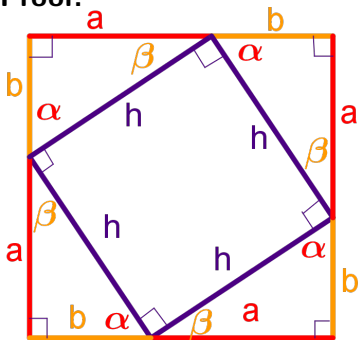
We get 4 congruent triangles

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

The triangles are congruent

Continuing this process

We get 4 congruent triangles

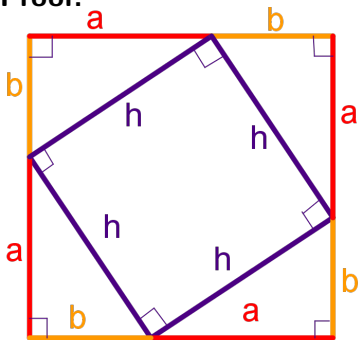
Recall: $A_{\Delta} = \frac{1}{2} \text{base} \cdot \text{height}$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



We label the angles α and β

We bring h^2 into the picture

To begin our next square, we extend b by length a

To find this angle, we use that $90 + \alpha + \beta = 180$

And $90 + \alpha + ? = 180$

We complete this triangle

The triangles share h, β, a

The triangles are congruent

Continuing this process

We get 4 congruent triangles

Recall: $A_{\Delta} = \frac{1}{2} \text{base} \cdot \text{height}$

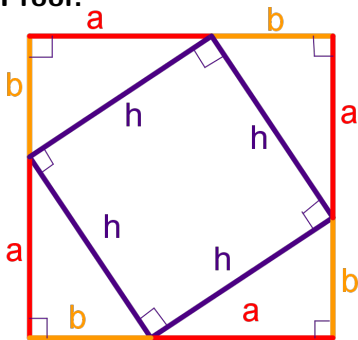
$$A_{\Delta} = \frac{1}{2} ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

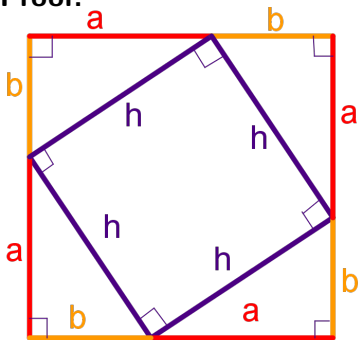
$$A_{\Delta} = \frac{1}{2}ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

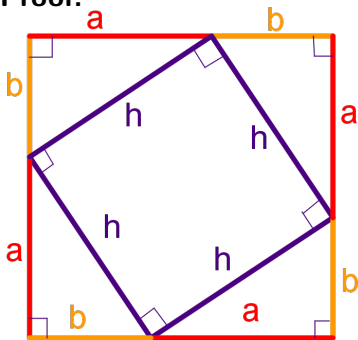
We can compute the area of the large square by computing the area of the **smaller square** and four triangles

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

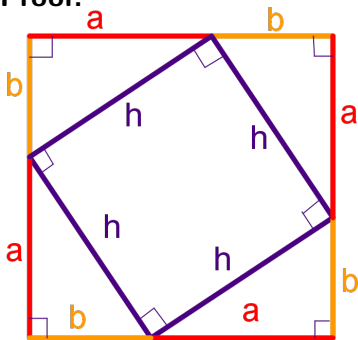
$$A = h^2 + 4 \cdot \frac{1}{2}ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

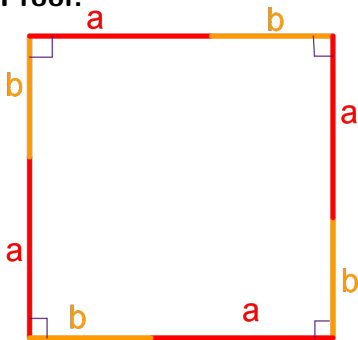
$$A = h^2 + 4 \cdot \frac{1}{2}ab = h^2 + 2ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

We can also compute the area of the large square by computing the area directly as a square with sides $a + b$

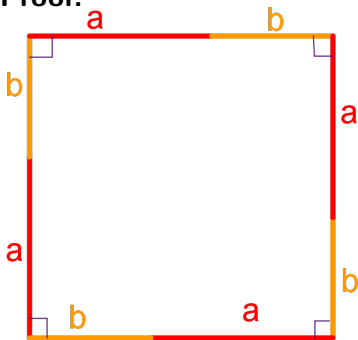
$$A = h^2 + 4 \cdot \frac{1}{2}ab = h^2 + 2ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

We can also compute the area of the large square by computing the area directly as a square with sides $a + b$

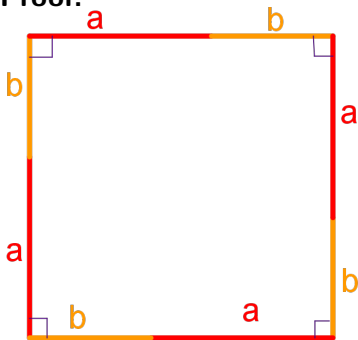
$$(a + b)^2 = A = h^2 + 4 \cdot \frac{1}{2}ab = h^2 + 2ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

► Recall how to square $(a + b)$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

We can also compute the area of the large square by computing the area directly as a square with sides $a + b$

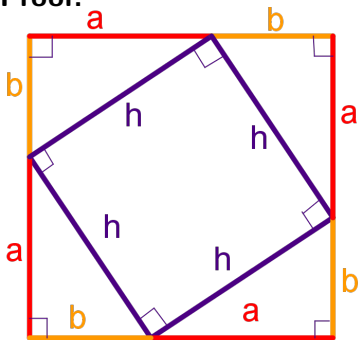
$$a^2 + b^2 + 2ab = (a + b)^2 = A = h^2 + 4 \cdot \frac{1}{2}ab = h^2 + 2ab$$

Pythagorean Theorem

Pythagorean Theorem states that for right triangles:
if a and b are the legs' lengths and h the hypotenuse, then

$$h^2 = a^2 + b^2$$

Proof:



Summary:

We get 4 congruent triangles

$$A_{\Delta} = \frac{1}{2}ab$$

► Recall how to square $(a + b)$

We can compute the area of the large square by computing the area of the **smaller square** and four triangles

We can also compute the area of the large square by computing the area directly as a square with sides $a + b$

$$a^2 + b^2 + \cancel{2ab} = (a + b)^2 = A = h^2 + 4 \cdot \frac{1}{2}ab = h^2 + \cancel{2ab}$$