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Since log_3 is a one-to-one function, with inverse 3^x, we can "undo" the *log* on each side by raising 3 to that power.

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$$3^{log_3\left(\frac{x+6}{x+2}\right)} = 3^{log_3(x)}$$

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Which is messy! But reduces nicely because the functions are inverses!

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Alternatively: the inputs of log_3 are equal because log_3 is one-to-one

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Since $log_3(-3)$ is undefined, $x = -3$ is not a solution

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Conc: The solution to $log_3(x+6) - log_3(x+2) = log_3(x)$ is $x = 2$