Recall: For a linear function (or line), the rate of change is called the slope and it is always the same between any two points.

For any function, y=f(x), the average rate of change between two points (x_1,y_1) and (x_2,y_2) is calculated the same as slope:

=

Note: The units of rate of change, average rate of change, and slope are all the same and are

Example 1 (Physics): Suppose that the height of a calculator thrown into the air is given by *s(t)*. The values of the height *s(t)* are given by the table:

t (sec)	0	1	2	3	4
s(t) (ft)	4	63	90	85	48

Find the average rate of change of position with respect to time, which is called the average velocity, on each interval.

Example 2: Find the average rate of change of $y = f \; x \; = x^2$ on the three intervals:

- (x = -1 to x=0)
- (*x = 0* to *x = 1*)
- (*x = 1* to *x = 2)*

-

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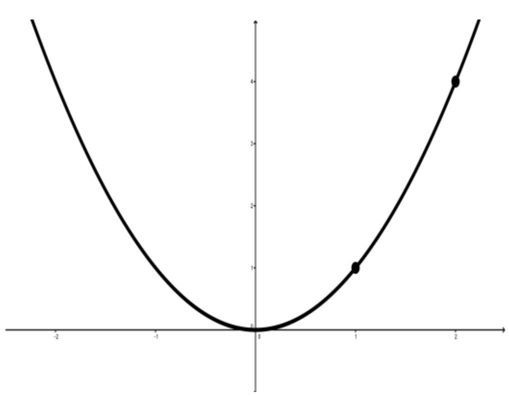

Example 1 (Physics): Suppose that the height of a calculator thrown into the air is given by *s(t)*. The values of the height *s(t)* are given by the table:

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Approximate the velocity at t=1

Approximation 1:

$v(1) \approx$

Approximation 2:

$v(1) \approx$

Approximation 3:

$v(1) \approx$

In general, the best and quickest way to approximate the instantaneous velocity is

t (sec)	0	1	2	3	4
s(t) (ft)	4	63	90	85	48
v(t)					

t (sec)	0	0.5
s(t)	4	37.5

$v\ 0\ \approx$

t (sec)	0	0.1
s(t)	4	11.34

$v\ 0\ \approx$

t (sec)	0	0.01
s(t)	4	4.7484

$v\ 0\ \approx$

t (sec)	0	0.001
s(t)	4	4.074984

$v\ 0\ \approx$

t (sec)	0	0.0001
s(t)	4	4.00749984

$v\ 0\ \approx$

Δt	Approx. of $v(0)$
1	59
.5	67
.1	73.4
.01	74.84
.001	74.984
.0001	74.9984
.00001	74.99984

$v(0) \approx$

As Δt goes to zero

$\Delta t \to 0$

$v\ 0\ =$

$s(t) = position; v(t) = velocity$

$v(t) =$ (instantaneous) rate of change of position

$$t = a$$

$$v(a) = \lim_{\Delta t \to o} \frac{s(a + \Delta t) - s(a)}{\Delta t}$$

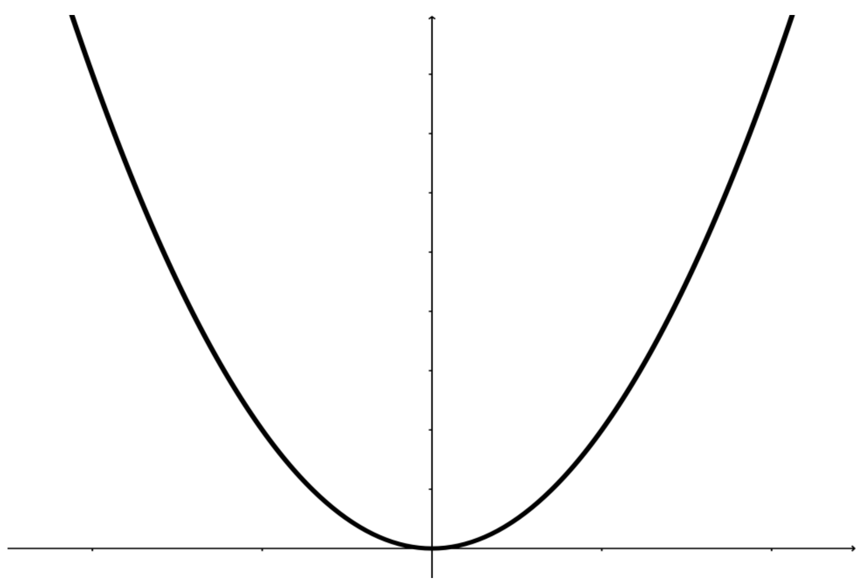
In general: The instantaneous rate of change of $f(x)$ at $x = a$ is given by:

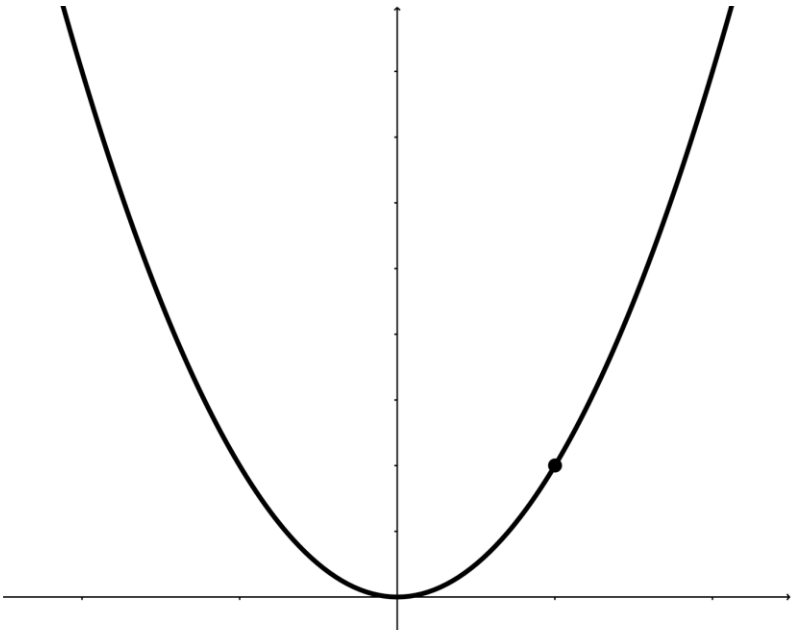
This is called the derivative of $f(x)$ at $x = a$, and is written as $f'(a)$.

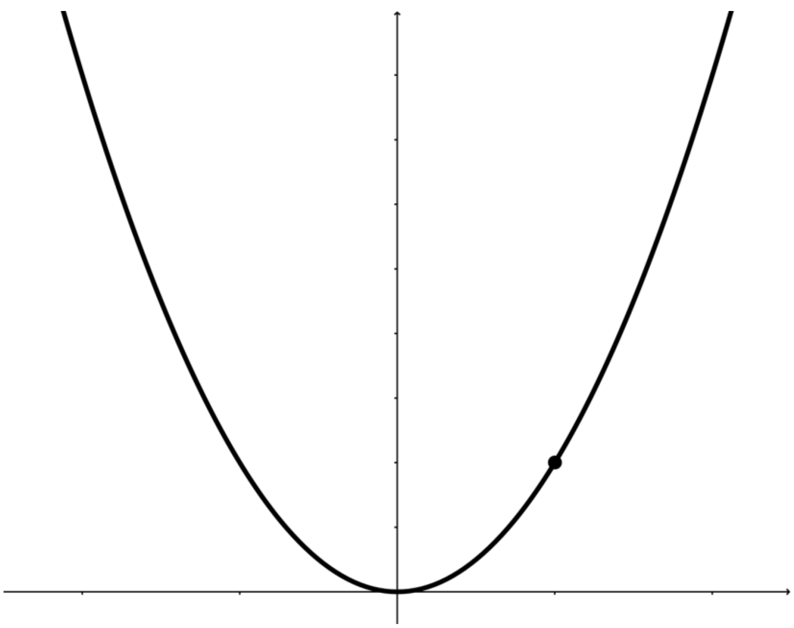
$f'(a) =$

Example: $f(x) = x^2$; find $f'(1)$

Δx	**Approx.** $f'(1)$







The derivative $f'(a)$ is:

Recall: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Example: $f(x) = 2^x$

Find $f'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$

Δx	**Approx. of $f'(1)$**

Recall: The derivative of the function $f(x)$ at $x = a$ is:

$$f'(a) = \lim_{\Delta x \to o} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

The derivative $f'(a)$ can be interpreted as:

- the rate of change of $f(x)$ at $x = a$
- the slope of line tangent to $f(x)$ at $x = a$
- the slope of curve at $x = a$

Example: Find the equation of the line tangent to the curve $y = x^2$ at $x = 1$.

Slope:

Point: