Recall: For a linear function (or line), the rate of change is called the slope and it is always the same between any two points.

For any function, y=f(x), the average rate of change between two points (x1,y1) and (x2,y2) is calculated the same as slope:

Average rate of change

between x = x1 and x=x2

=

Note: The units of rate of change, average rate of change, and slope are all the same and are

Example 1 (Physics): Suppose that the height of a calculator thrown into the air is given by *s(t)*. The values of the height *s(t)* are given by the table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t (sec) | 0 | 1 | 2 | 3 | 4 |
| s(t) (ft) | 4 | 63 | 90 | 85 | 48 |

Find the average rate of change of position with respect to time, which is called the average velocity, on each interval.

Average velocity between =

t = 0 and t =1

Average velocity between =

t = 1 and t =2

Average velocity between =

t = 2 and t =3

Average velocity between =

t = 3 and t =4

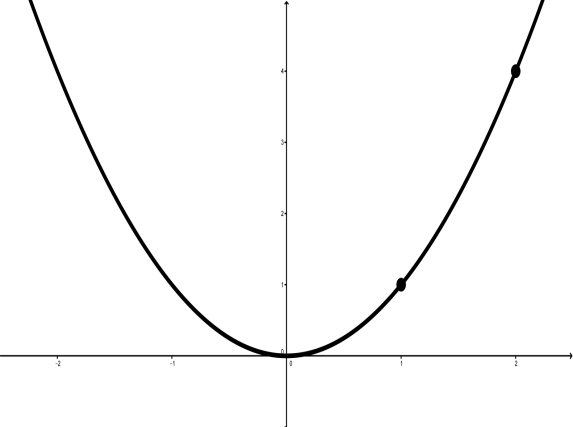
Example 2: Find the average rate of change of on the three intervals:

* (x = -1 to x=0)
* (*x = 0* to *x = 1*)
* (*x = 1* to *x = 2)*

Average rate of change between x = -1and x=0

Average rate of change between x = -1and x=0

Average rate of change between x = -1and x=0



Example 1 (Physics): Suppose that the height of a calculator thrown into the air is given by *s(t)*. The values of the height *s(t)* are given by the table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t (sec) | 0 | 1 | 2 | 3 | 4 |
| s(t) (ft) | 4 | 63 | 90 | 85 | 48 |

Approximate the velocity at t=1

Approximation 1:

Approximation 2:

Approximation 3:

In general, the best and quickest way to approximate the instantaneous velocity is

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t (sec) | 0 | 1 | 2 | 3 | 4 |
| s(t) (ft) | 4 | 63 | 90 | 85 | 48 |
| v(t) |  |  |  |  |  |

|  |  |  |
| --- | --- | --- |
| t (sec) | 0 | 0.5 |
| s(t) | 4 | 37.5 |

|  |  |  |
| --- | --- | --- |
| t (sec) | 0 | 0.1 |
| s(t) | 4 | 11.34 |

|  |  |  |
| --- | --- | --- |
| t (sec) | 0 | 0.01 |
| s(t) | 4 | 4.7484 |

|  |  |  |
| --- | --- | --- |
| t (sec) | 0 | 0.001 |
| s(t) | 4 | 4.074984 |

|  |  |  |
| --- | --- | --- |
| t (sec) | 0 | 0.0001 |
| s(t) | 4 | 4.00749984 |

|  |  |
| --- | --- |
|  | **Approx.**  **of** |
| 1 | 59 |
| .5 | 67 |
| .1 | 73.4 |
| .01 | 74.84 |
| .001 | 74.984 |
| .0001 | 74.9984 |
| .00001 | 74.99984 |

As goes to zero

;

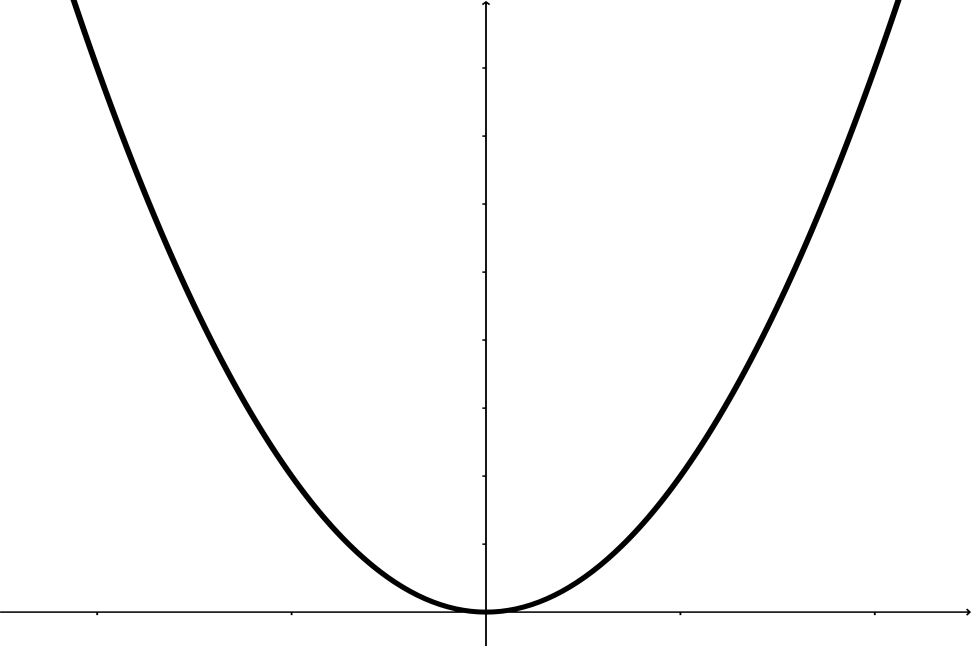
(instantaneous) rate of change of position

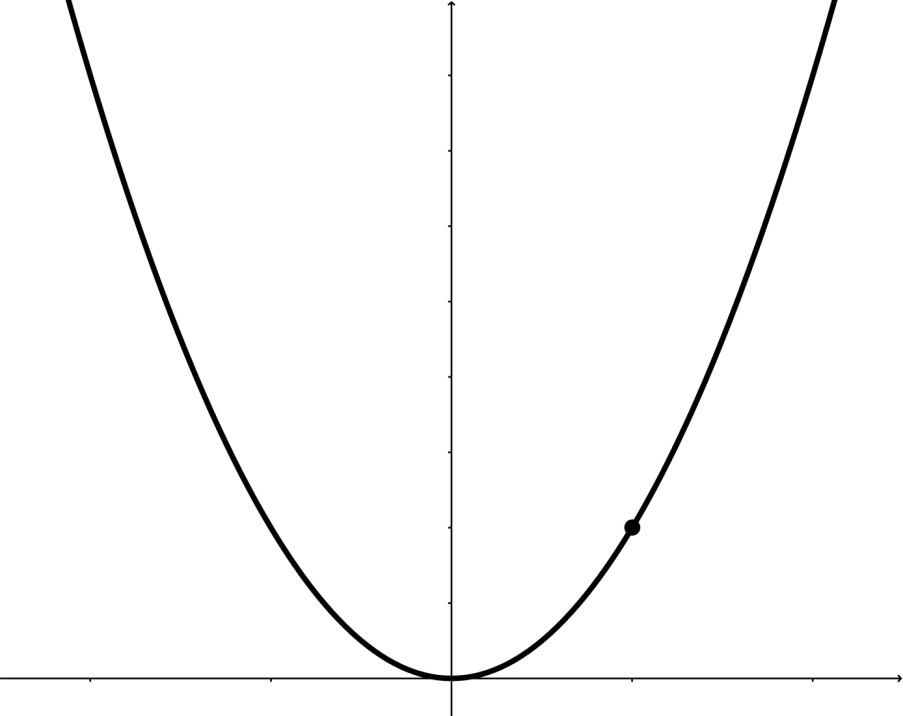
In general: The instantaneous rate of change of at is given by:

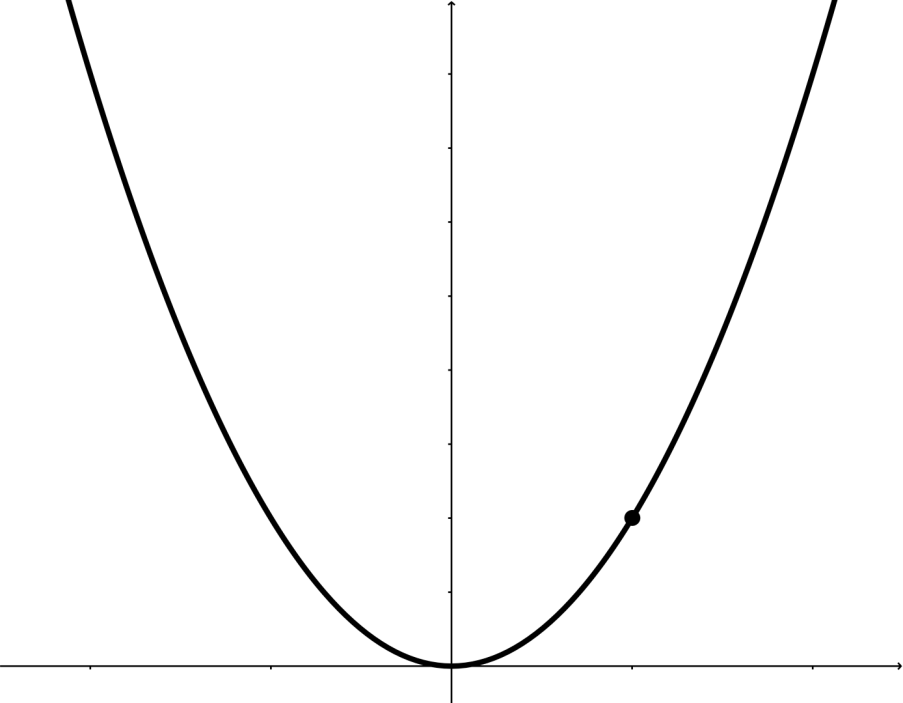
This is called the derivative of at , and is written as .

|  |  |
| --- | --- |
|  | **Approx.** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Example: ; find







The derivative is:



Recall:

Example:

Find

|  |  |
| --- | --- |
|  | **Approx.**  **of** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Recall: The derivative of the function at is:

The derivative can be interpreted as:

* the rate of change of at
* the slope of line tangent to at
* the slope of curve at

Example: Find the equation of the line tangent to the curve at .

Slope:

Point:

