Recall: If we borrow $P_o = P(0)$ dollars for t years with an annual interest rate r with interest compounded continuously then the amount owed is given by:

$$P(t) = P_o \cdot e^{rt}$$

Example: When will we owe $24000 if we borrow $15000 with a 6% interest rate compounded continuously?

Define: the function $f(x) = \ln(x)$ is the inverse of the function $y = e^x$ and is called the natural logarithm.

That is: $\ln(e^k) =$

And $e^{\ln(k)} =$

Finish Example:

Define: the function $f(x) = \ln(x)$ is the inverse of the function $y = e^x$ and is called the natural logarithm.

That is: $\ln(e^k) = k$ for any value of k.

And $e^{\ln(k)} = k$

Additional properties of logarithms:

1. $\ln(A \cdot B) =$

2. $\ln\left(\frac{A}{B}\right) =$

3. $\ln(a^p) =$

Example 1:

Solve $2^x = 5$

Example 2:

Solve $3 \cdot 2^x = 5$

Example 3:

Solve $2^x = 3^{(x+1)}$

Example 4: The radio-active element Cobalt-60 loses 13.5% of its mass each year.

If we start with 250g of Cobalt-60, how long until we have half as much as we started?

Example 4b: The radio-active element Cobalt-60 loses 13.5% of its mass each year.

How long until there is half as much Cobalt-60 as at the start?

Def: The amount of time it takes for a substance to decay exponentially to half its original amount is called the half-life.

Def: The amount of time it takes for a substance to grow exponentially to twice its original amount is called the doubling-time.

Example: Find the doubling time for the bacteria colony whose population we modelled earlier by:

$$P(t) = P_o \cdot 1.5^t$$

$t =$# of days; $P(t) =$ population after t days