Suppose we borrow $100,000 from the bank to start a company, with the plan to pay it back in one year.

How much will we have to pay back?

If the bank charges us a 10% interest rate, then how much do we have to pay the bank in one year?

Suppose the bank allows us to borrow the money for a second year as long as we still pay the 10% interest rate.

How much will we owe after 2 years?

Paying interest on interest is called compound interest.

How much will we owe if we borrow $100000 for 3 years with an annual interest rate of 10%?

The amount of money we owe is called the principle.

$P(t) =$ amount of money owed (principle) after t years.

$t =$ number of years the money is borrowed

$P(0) =$

$P(1) =$

$P(2) =$

$P(3) =$

$P(t) =$

In general: If we borrow $P_o = P(0)$ dollars for t years with an annual interest rate r, then the amount owed is:

An exponential function is a function of the form:

$$P(t) =$$

Where P_o and a are constants.

t	0	1	2	3
P(t)	100000	110000	121000	133100

Average rate of change:

From $t = 0$ to $t = 1$:

From t =1 to $t = 2$:

From $t = 2$ to t =3:

Relative change:

From $t = 0$ to $t = 1$:

From t =1 to $t = 2$:

From $t = 2$ to t =3:

For the function $P(t) = 100000 \cdot 1.1^t$; relative change is:

$$P(t) = P_o a^t$$

$P(0) = P_o =$ original amount

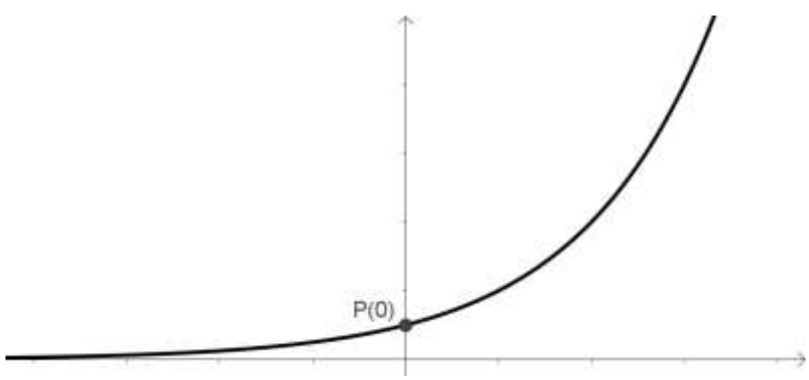
$a = 1 + r$ where $r =$ relative change

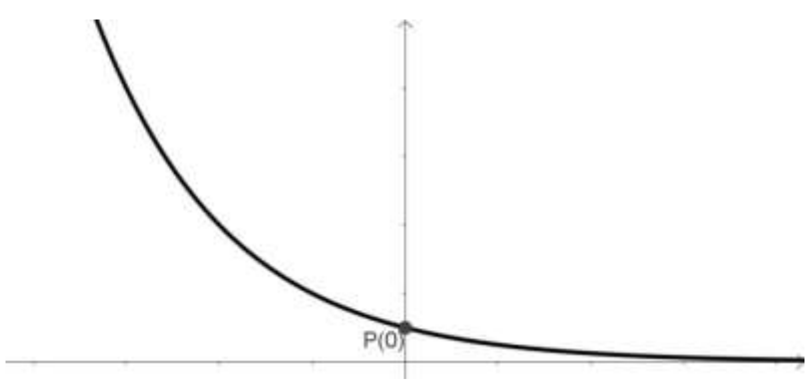
If $a > 1$ then $r > 0$ and we have positive (relative) change, so the exponential function is increasing.

In this case, we say r is the exponential *growth* rate.

If $a < 1$ then $r < 0$ and we have negative (relative) change, so the exponential function is decreasing.

In this case, we say r is the exponential *decay* rate.





Populations:

Recall: When we looked at the populations of Williamsburg, MA and New York City it was useful to talk about *relative* (or *percent*) *change*.

If a population has constant relative change (which means it grows the same percent each year) then we can use exponentials to model population growth.

Example: Suppose that we are growing a bacteria colony on a petri dish.

t (days)				
P(t)				

Relative change $t = 0$ to $t = 1$ =

Relative change $t = 1$ to $t = 2$ =

Relative change $t = 2$ to $t = 3$ =

Example: Suppose that we are growing a bacteria colony on a petri dish. Find a model to describe the population.

Example 2: Using the model, estimate the population of bacteria after 10 days.

Recall: If we borrow $P_o = P(0)$ dollars for t years with an annual interest rate r, then the amount owed, P, is:

$$P(t) = P_o \cdot (1 + r)^t$$

Back to Example: Suppose we borrow $\$100000$ with a 10% annual interest rate. Then the amount we have to pay back after t years is:

$$P(t) = 100000 \cdot (1 + .1)^t$$

$$P(t) = 100000 \cdot (1.1)^t$$

What if the bank wants to collect interest after each month instead of waiting a year?

$P\left({}^1/_{12}\right) =$ amount owed after one month (${}^1/_{12}$ of a year)

$P\left({}^1/_{12}\right) =$

How much is owed after two months? $t = {}^2/_{12}$

$P\left({}^2/_{12}\right) =$

Similar calculation for the amount owed after 3 months shows:

$P\left({}^3/_{12}\right) =$

Back to Example: Suppose we borrow $\$100000$ with a 10% annual interest rate, interest collected each month.

$P\left({}^{1}/_{12}\right) =$

$P\left({}^{2}/_{12}\right) =$

$P\left({}^{3}/_{12}\right) =$

$P(1) =$

$P(2) =$

$P(t) =$

Example: How much money do we owe after 3 years?

In the above example, interest is collected each month, which is 12 times peryear.

If we collect interest each day, or 365 times per year, then the new formula is:

In general: If we borrow $P_o = P(0)$ dollars for t years with an annual interest rate r with interest collected n times per year, then the amount owed, P, is:

P(t)=

What happens as $n \to \infty$?

In general: If we borrow $P_o = P(0)$ dollars for t years with an annual interest rate r with interest compounded continuously then the amount owed is given by:

P(t)=

Example: How much money will we owe after 5 years if we borrow $15000 with a 6% interest rate compounded continuously?

Recall: The amount owed on a P_o dollar loan for t years with interest rate r, compounded n times per year is:

$$P(t) = P_o \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

If we compound continuously, then:

$$P(t) = P_o \cdot e^{rt}$$

Example 1: Suppose you take out $20000 in student loans to graduate college. Bank A offers a 6% interest rate compounded quarterly. Bank B offers 5.75% interest rate compounded continuously. Which bank would you choose?

Example 2: Suppose you run a furniture store that offers a couch for $2000 with No Interest if Paid in Full in 1 Year!*

*If not paid in full, then interest must be paid on the full purchase price ($2000) for the entire year at 27.99% compounded monthly (as credit cards often are).

How much more money will our store make on each customer that does not pay their account off in full in the one year?

Example 3: Suppose you run a chain of furniture stores that does $5 million in sales each year on customers that do not pay off their account in full by the end of the year.

How much money will your store make in interest?

Recall: An exponential function is of the form

$$P(t) = P_o a^t$$

$a = 1 + r$ where $r =$ relative change

If $a > 1$, then there is exponential growth. This is useful for modelling populations, loans, amongst other applications.

If $a < 1$, then there is exponential decay.

Example 1: The amount of caffeine in the average person's bloodstream decreases by 11% each hour.

If someone drinks a coffee with 120mg of caffeine, how much caffeine will be in their bloodstream 6 hours later?

Example 2: Radio-active elements decay exponentially as well. In particular, the radio-active element Cobalt-60 loses 13.5% of its mass each year.

If we have a sample of Cobalt-60 that has a mass of 250g, how much will be left after 2 years?