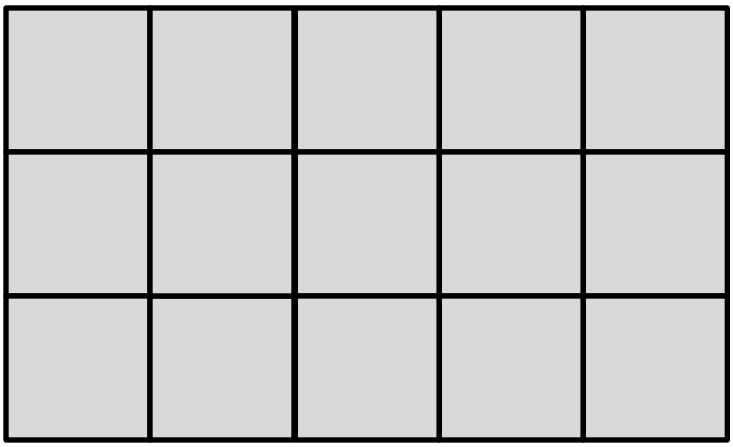
Visualization of Multiplication

$3 \cdot 5 = 15$

Visualization:



Conclusion: We can visualize multiplication of $a \cdot b$ as a rectangle of height a and width b.

Example: If we travel at a velocity of $10\ ft/sec$ for 3 seconds, how far did we travel?

Solution:

$$Distance = velocity \cdot time$$

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)				
(ft/s)				

How far did the runner travel from $t = 0$ to $t = 2$?

Overestimate:

Underestimate:

How far did the runner travel from $t = 2$ to $t = 4$?

Overestimate:

Underestimate:

How far did the runner travel from $t = 4$ to $t = 6$?

Overestimate:

Underestimate:

How far did the runner travel from $t = 0$ to $t = 6$?

Overestimate:

Underestimate:

How do we get a better estimate?

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)							
(ft/s)				**11**			

How far did the runner travel from $t = 0$ to $t = 6$?

Overestimate:

Underestimate:

Note: If we want our approximation to be a number, average the over- and under-estimates.

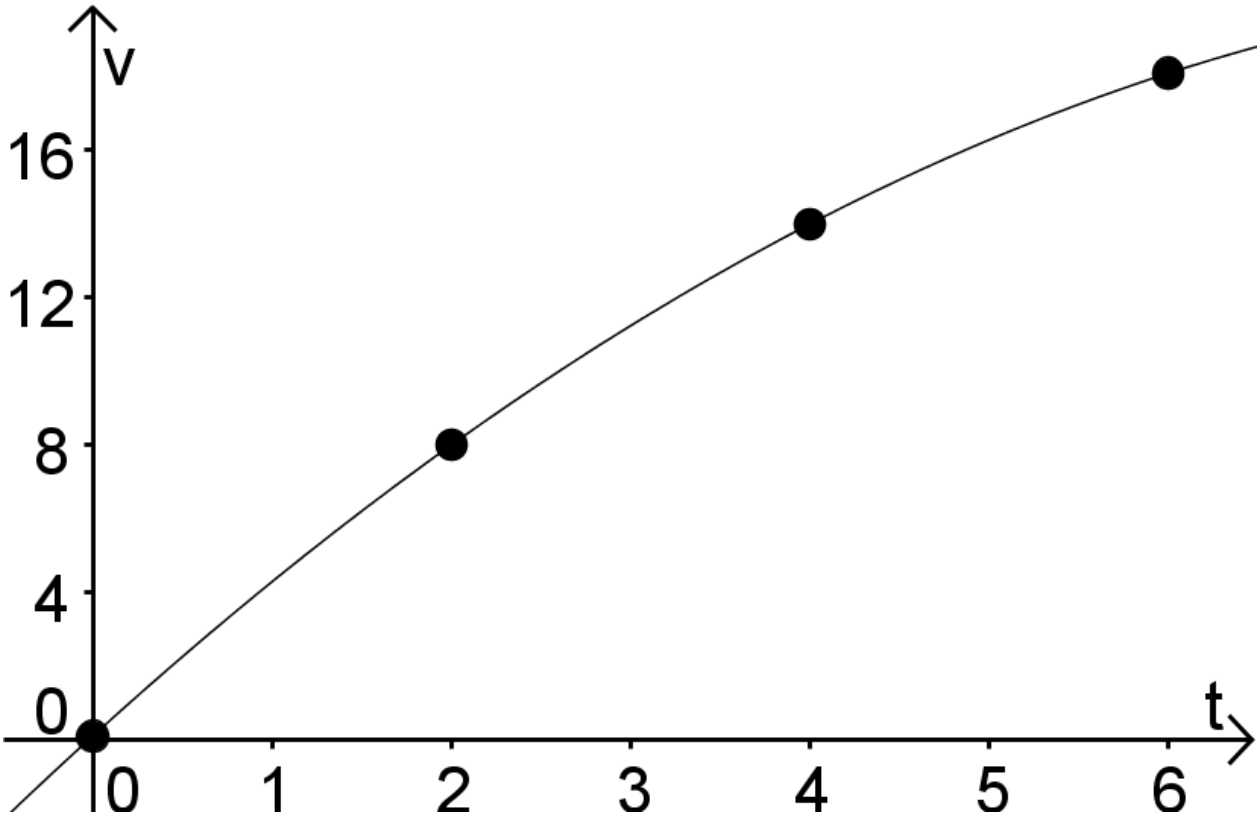
$$Distance\ Travelled \approx$$

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)				
(ft/s)				

How far did the runner travel from $t = 0$ to $t = 6$?

Overestimate: $8 \cdot 2 + 14 \cdot 2 + 18 \cdot 2 = 80$



Visualization of Multiplication:

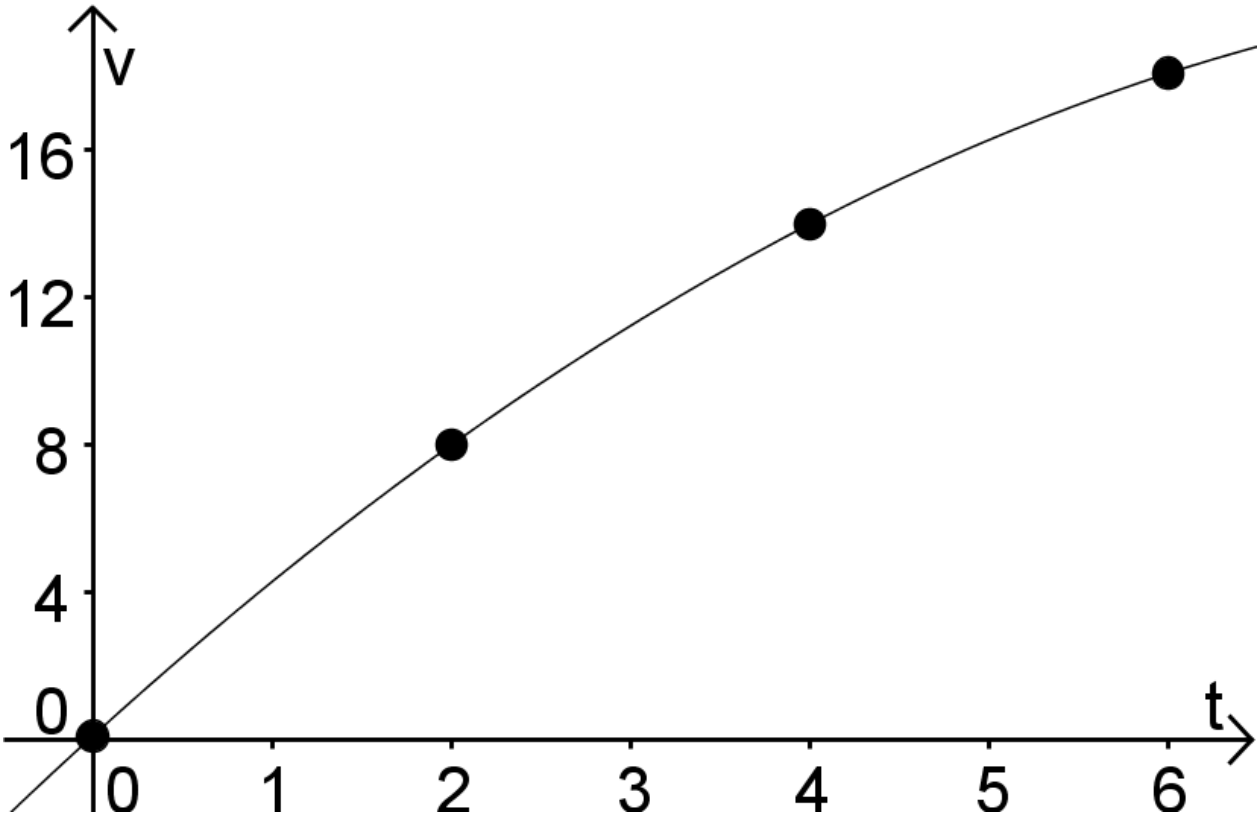
Distance Travelled from Distance Travelled from Distance Travelled from

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)				
(ft/s)				

How far did the runner travel from $t = 0$ to $t = 6$?

Underestimate: $0 \cdot 2 + 8 \cdot 2 + 14 \cdot 2 = 44$



Visualization of Multiplication:

Distance Travelled from

Distance Travelled from

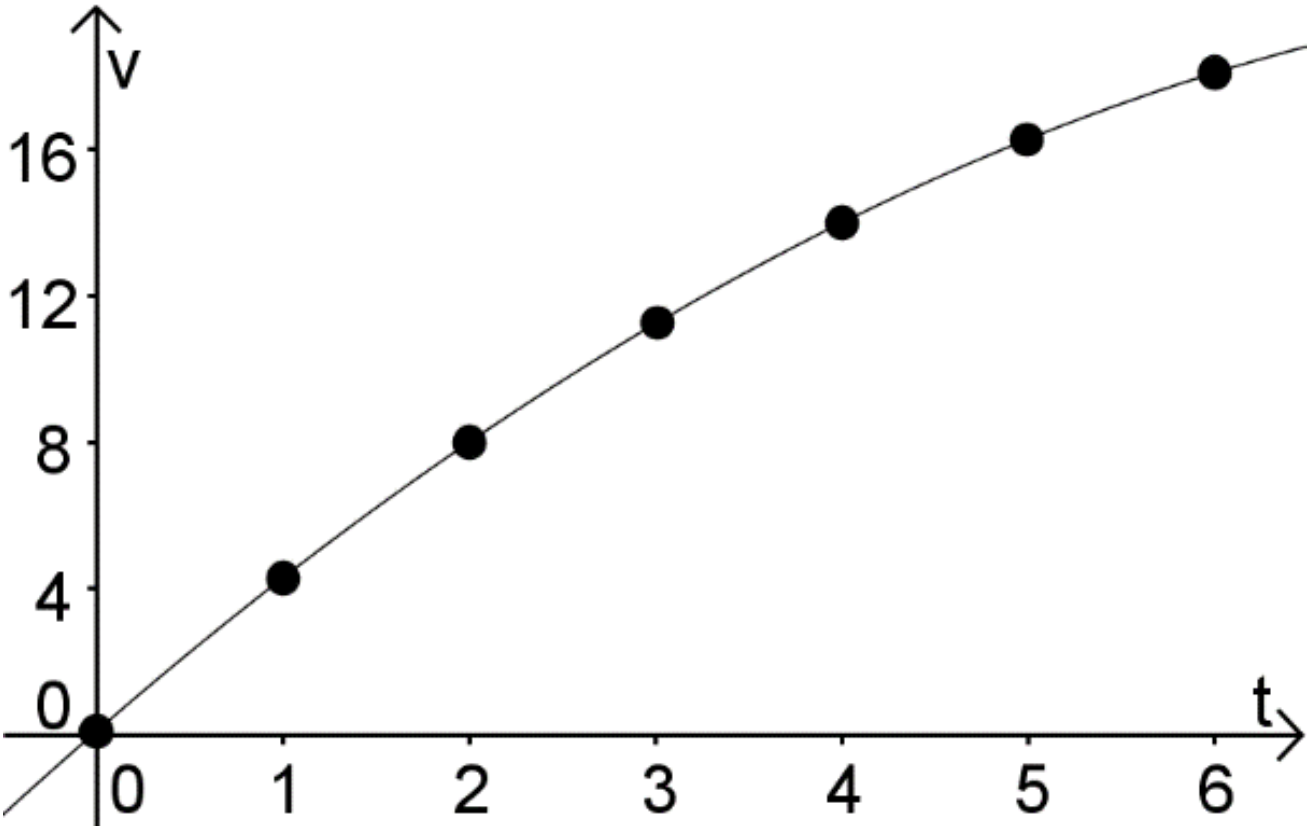
Distance Travelled from

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)							
(ft/s)				**11**			

How far did the runner travel from $t = 0$ to $t = 6$?

Overestimate: $4.5 \cdot 1 + 8 \cdot 1 + 11 \cdot 1 + 14 \cdot 1 + 16.5 \cdot 1 + 18 \cdot 1 = 72$



Visualization of Multiplication:

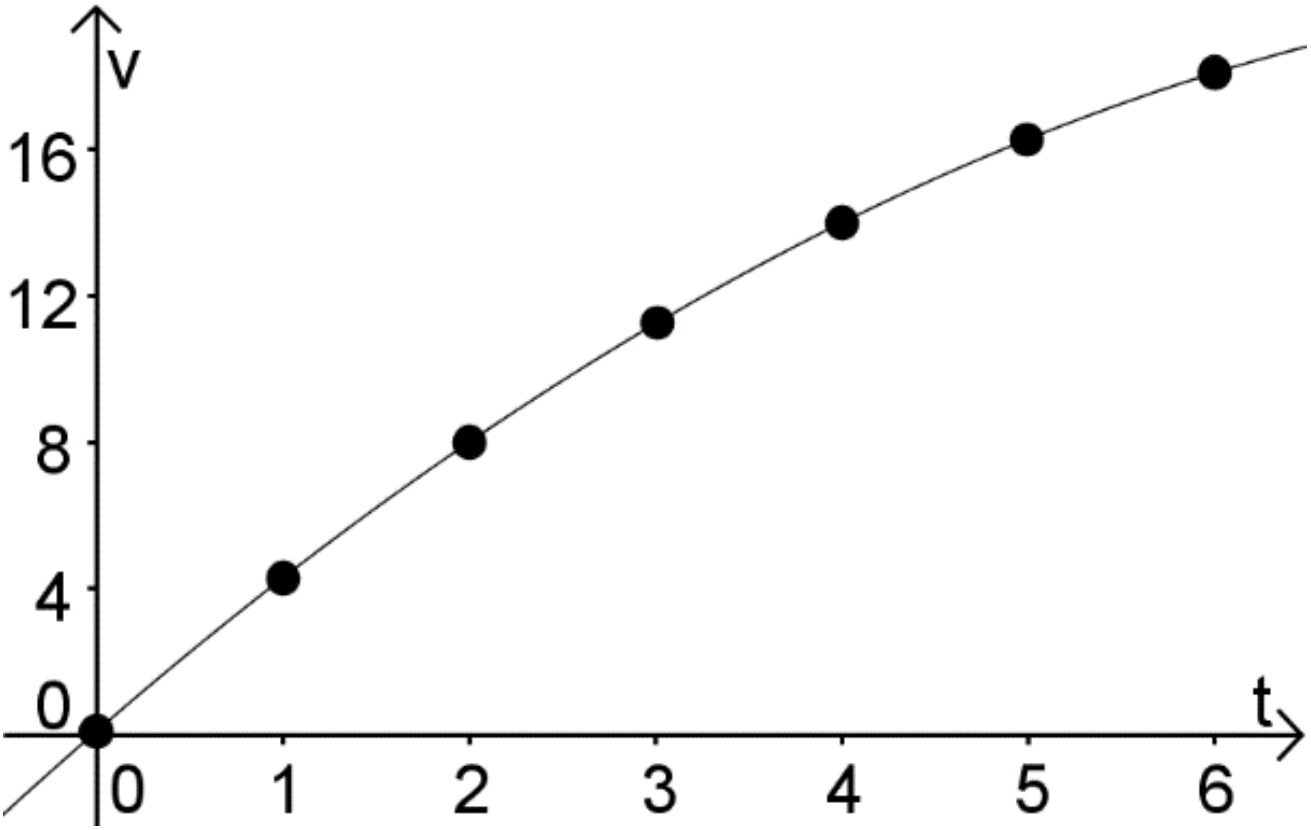
Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from

Example: Velocities (in ft/sec) of a runner starting a race are:

(sec)							
(ft/s)				**11**			

How far did the runner travel from $t = 0$ to $t = 6$?

Underestimate: $0 \cdot 1 + 4.5 \cdot 1 + 8 \cdot 1 + 11 \cdot 1 + 14 \cdot 1 + 16.5 \cdot 1 = 54$



Visualization of Multiplication:

Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from Dist Travelled from

How far did the runner travel from $t = 0$ to $t = 6$?

Overestimate: The runner travelled at most 72 feet in 6 seconds.

Underestimate: The runner travelled at least 54 feet in 6 seconds.

How do we get a better estimate?

Notes from Graphs:

1.

2.

3.

$$s\ t\ = position; v\ t\ = velocity$$

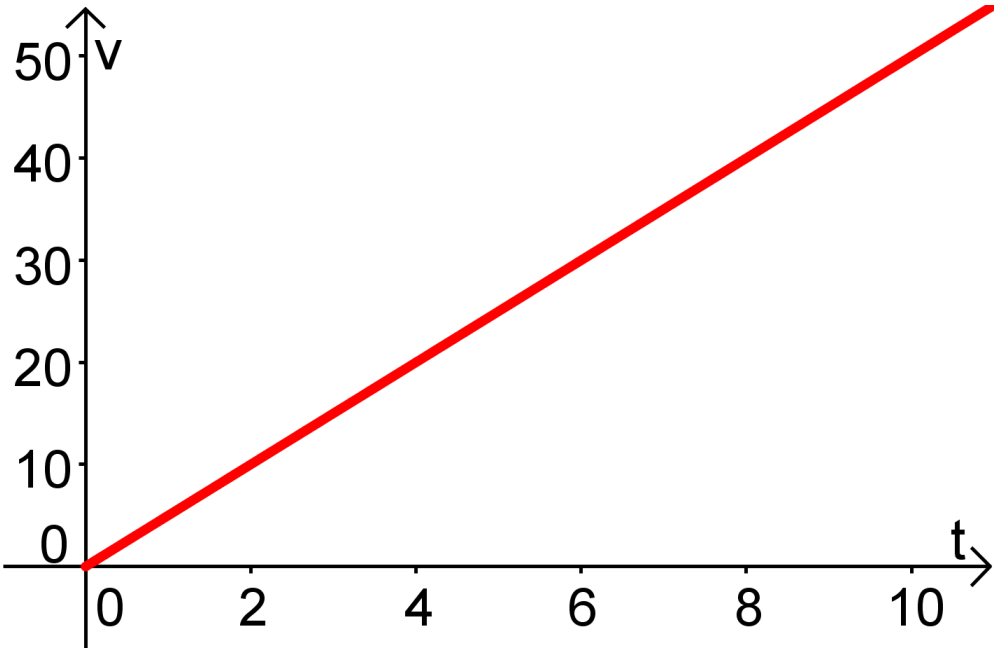
$$v\ t\ = s'(t)$$

Example: Suppose that a car's velocity, given is feet/sec, is:

$$v\ t\ = 5t$$

How far does the car travel in the first 10 seconds?

Solution:



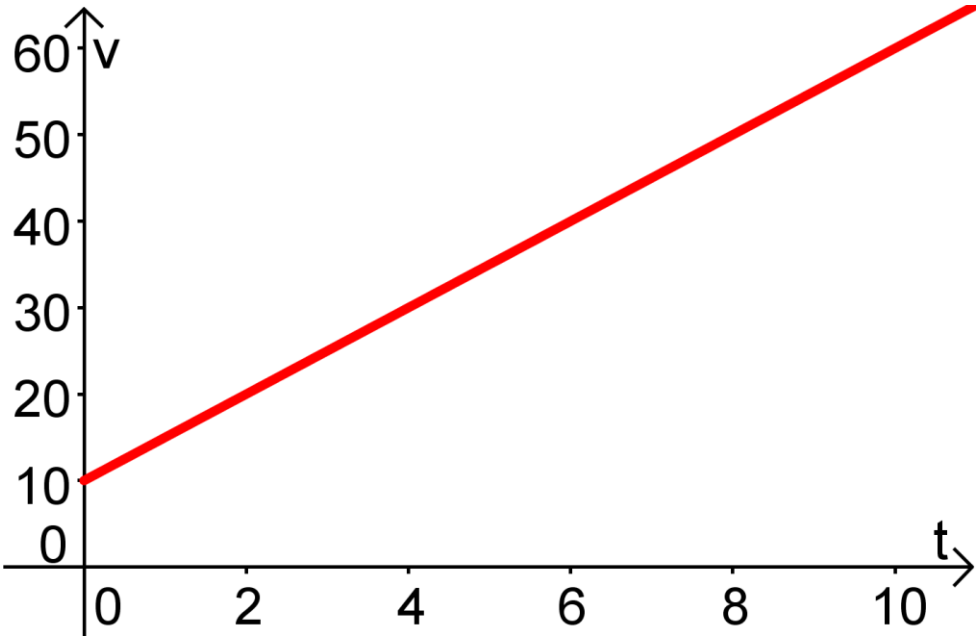
Conclusion:

Example 2: Suppose that a car's velocity, given is feet/sec, is:

$$v\ t\ = 5t + 10$$

How far does the car travel in the first 10 seconds?

Solution:



Conclusion: