

MATH 331 HOMEWORK 2, SEC 2.3-2.8

Problem 1: A 250 gallon tank initially contains 160 gallons of pure water. Water with a concentration of c pounds of salt per gallon is added to the tank at 2 gal/min, and the resulting solution leaves at a rate of 1 gal/min. What concentration c should be used so that the tank will have a concentration of $\frac{1}{4}$ pounds per gallon once it is full with 250 gallons of water in the tank?

Problem 2: Consider the following equation for a certain population of squirrels given by $P(t)$ (t is measured in years):

$$\frac{dP}{dt} = 2.08 \cdot P \left(1 - \frac{P}{8} \right) \left(\frac{P}{4} - 1 \right)$$

- Find all the equilibrium points of the equations.
- Draw the phase line and determine the stability of each equilibrium points.
- Graph the tP -plane with the 6 solution curves corresponding with the equilibrium solutions and the solutions with initial conditions $P(0) = 3$, $P(0) = 5$ and $P(0) = 10$.

Problem 3: The amount of money in an account, $S(t)$, which starts with $\$S_0$ and grows with a continuous annual interest rate of r and continuous deposits of $\$D$ per year can be modeled by the initial value problem:

$$\frac{dS}{dt} = r \cdot S + D \quad S(0) = S_0$$

- Suppose that a student, upon graduation, inherits $\$3000000$, which they put into an investment account at age 22 and earns a continuous annual interest rate of 6% on their investments.

The student wishes to have $\$24000000$ at the time that they retire. How much money can they continuously *withdraw* each year and still have $\$24000000$ at the time that they retire at age 62?

Note: In the case of withdrawals, $D < 0$.

- Find the amount D_2 that someone needs to make in yearly contributions starting at age 22 to retire with the same amount of money age 62, assuming the same 6% continuous annual interest rate on their investment but with an initial investment of $\$0$.

Problem 4: Find all functions $M(x, y)$ such that the equation $M(x, y) + (3x^3y^2 - x \sin(y)) \frac{dy}{dx} = 0$ is exact.