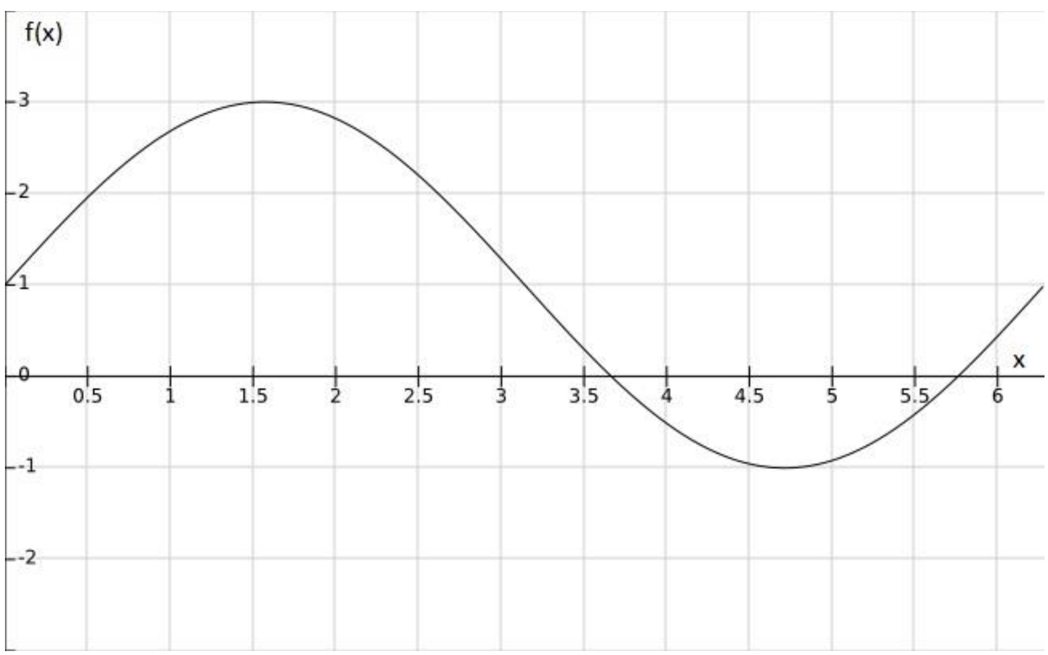
Average Rate of Change

Calculus is the study of change.

What we know at this point is how to measure the change of a linear function.

Remember Slope of a line:

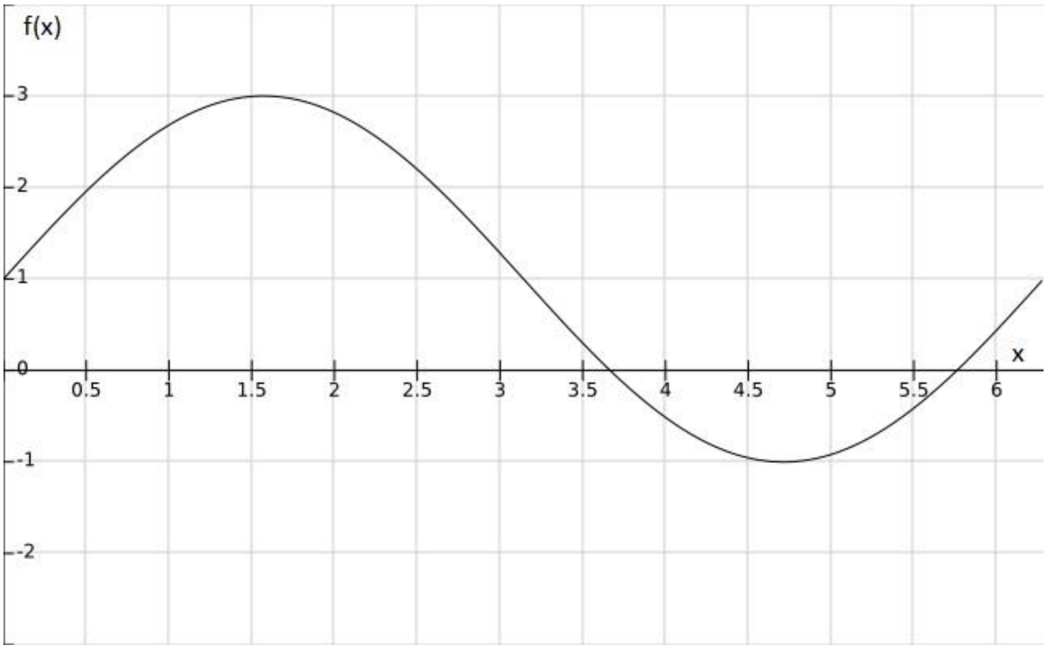
What if we want to measure the change of a curved function?



We can calculate how much a function changes on average over an interval by computing the change in the dependent variable over the change in the independent variable.

AVERAGE RATE OF CHANGE FORMULA:

Graphical representation of the average rate of change.



When using a line to approximate the average rate of change between two points on a non-linear function that line is called the ___________________________.

Note: the **average rate of change** of a linear function is the slope, and a function is linear if the average rate of change is the same on all intervals.

EXAMPLE 1: Find the average rate of change between $x = 1$ and $x = 4$ for the function

$y = f(x) = x^2 + 3x - 1.$

The average rate of change tells us how much a function changes on average. The word average is used because the rate of change may vary within the interval.

<u>EXAMPLE 2</u>: Suppose the table below is continuous and gives the data of a population, P, is thousands measured in years t, since 1970.

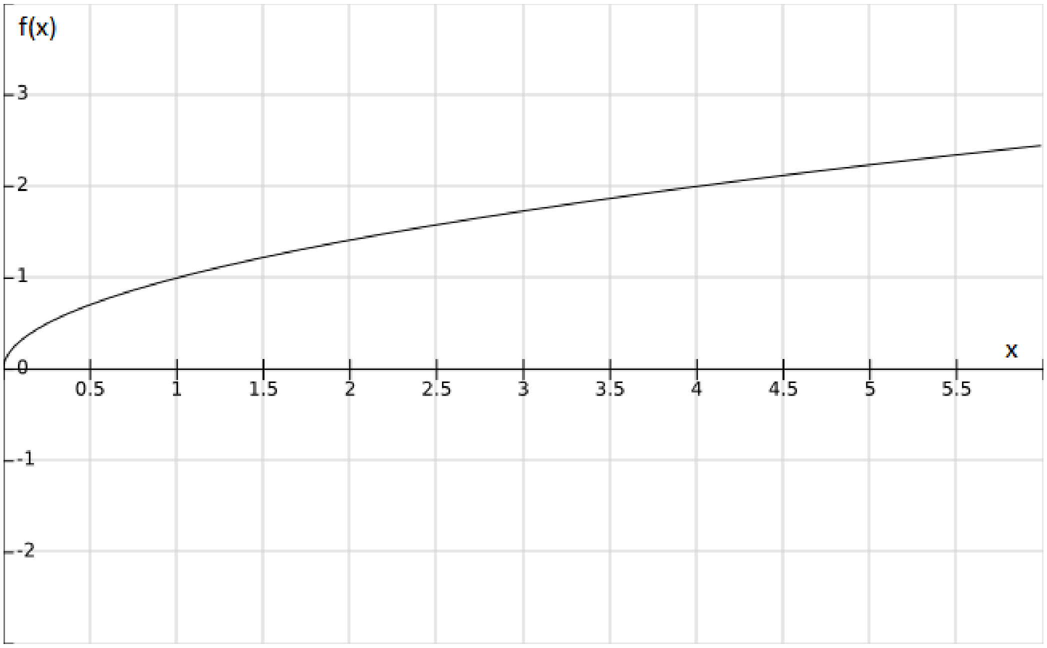
t	0	10	20	30	40
$P(t)$	60	65	73	85	100

1. What was the population in 1970?

2. What was the average rate of change between 1970 and 2000?

3. What was the average rate of change between 1980 and 2010?

EXAMPLE 3: Using the graph below find the average rate of change from $x = 0$ to $x = 4$.



Distance and Velocity

Distance:

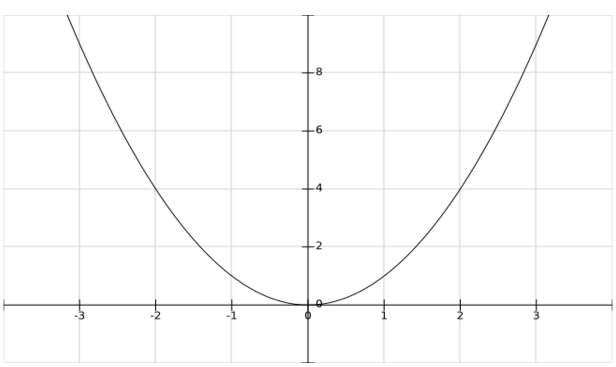
Velocity:

Estimating Velocity from Distance using average rate of change:

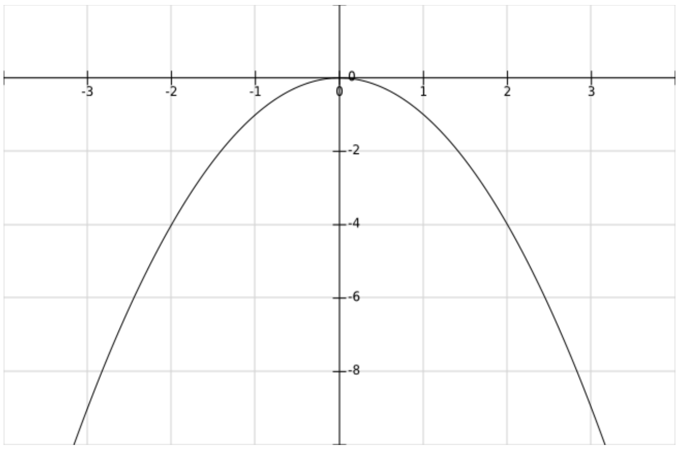
EXAMPLE 4: Suppose the position of an object falling off of a building can be found using the formula $S(t) = 2704 - 16t^2$, where position, $S(t)$, is measured in feet and time, t, is measured in seconds. First find the time when this object hits the ground. Then find the average velocity of the object from $t = 3$ and $t = 6$.

Concavity describes the bend in a function.

Concave up:



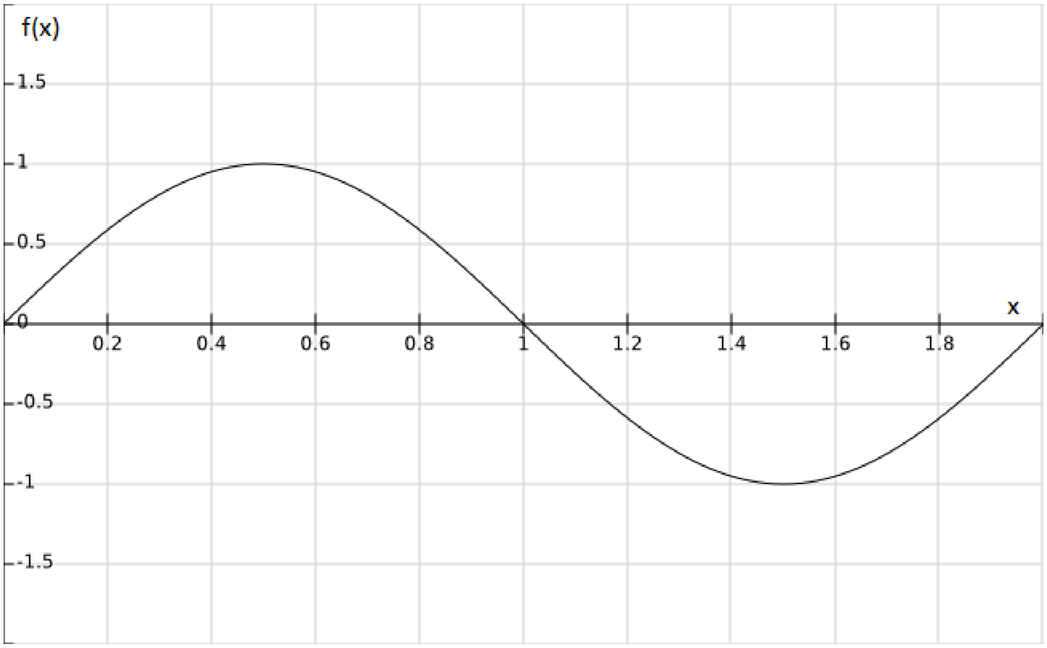
Concave down:



EXAMPLE 5: Assume that the function represented below in continuous. Determine where the following function is likely to be concave up and concave down.

x	0	2	4	6	8	10	12	14
$f(x)$	10	12	16	22	30	35	38	39

<u>EXAMPLE 6:</u> Using the graph below determine the intervals where this function is increasing, decreasing, concave up and concave down.



<u>Relative Change</u>

Consider: If you received a discount of $1 on a product, was that deduction a significant change?

Scenario 1: You are buying a car for $25,000 and the dealership offers to sell it to you for $24,999.

Scenario 2: You always but a medium iced coffee at Dunkin for $2.64. Today they have a promotion that a medium iced coffee costs $1.64.

In these two scenarios you have received a $1 discount; however one feels better than the other. Why?

Relative Change (aka percent change):

The <u>relative change</u> is a number, without units, often expressed as a percentage

EXAMPLE 1: Calculate the Relative Change for the scenarios presented above:

Scenario 1: You are buying a car for $25,000 and the dealership offers to sell it to you for $24,999.

Scenario 2: You always but a medium iced coffee at Dunkin for $2.64. Today they have a promotion that a medium iced coffee costs $1.64.

With relative change we use words like decreasing or discounted to express that the numerical change was negative. We use words like increasing or grew to express that the numerical change was positive.

EXAMPLE 2: Find the relative change for each of the following scenarios if price increases by $4.

Scenario 1: A gallon of milk that costs $4

Scenario 2: A Chromebook computer that costs $150.

EXAMPLE 3: Suppose that when a store sells a particular necklace for $60, they sell 65 per week. When they have their annual store sale they reduce the price by 25% and during that week they sell 110 necklaces. Find the relative change in weekly sales.

Ratio of Relative Changes (Elasticity):

EXAMPLE 3(cont.) : Suppose that when a store sells a particular necklace for $60, they sell 65 per week. When they have their annual store sale they reduce the price by 25% and during that week they sell 110 necklaces. Find the relative change in weekly sales.

Example 4: Suppose the cost of a pound of high end local ground coffee is normally $19.99. At this price 275 pounds are sold each month. When the coffee goes on sale for $14.99 during the month of April, 425 pounds are sold. Find the relative change in price. Find the relative change in monthly sales between the typical month and the sale month. Find the elasticity of this product.