Welcome to College Algebra

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For a long time (both individually and historically) these were enough numbers.

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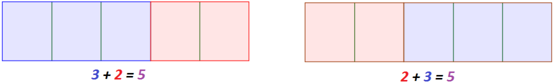
Of course: $2 + 3 = 5$



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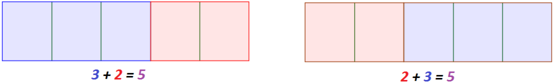

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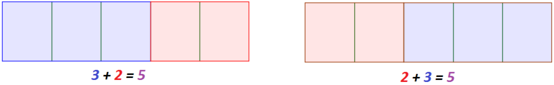

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And there is nothing special about the numbers 2 and 3 here.

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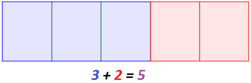

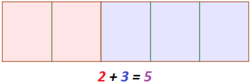

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Commutative Property of Addition: For any numbers A, B

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(Having the number 0 comes in handy here).

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There is something special about adding 0.

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The question becomes, how many are left for you?

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Starting with our 3 and removing 1, we see that $3 - 1 = 2$

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That is: $3 - 3 = 0$

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That is: $3 - 3 = 0$
3 is not special here, this is true of any number A:

$$A - A = 0$$

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Our first property about subtraction: For any number A:

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Does this work for Subtraction?

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Since Addition is nicer, let's stick with addition for now!

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Example 4: As we keep hunting and eating, we learn that our society needs 4 antelope to eat for a week

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$$3 + ? = 4$$

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Our numbers so far:

$$\{0, 1, 2, 3, ...\}$$

Example 4: As we keep hunting and eating, we learn that our society needs 4 antelope to eat for a week
We know that we can consistently catch 3 antelope.
How many does our (inconsistent) hunting partner need?
Since we know we add our numbers together, this is addition.
But different... Now we have:

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Like before, we are trying to figure out a number. But now, we want to be able to add the number that we don't know!

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This is where Algebra is born!
Because ? is not a very easy symbol to work with, we will call our unknown number x
We now have a name for our number x, but how to we find out what it is?!

Welcome to College Algebra

Our numbers so far:

$$\{0, 1, 2, 3, ...\}$$

$$3 + x = 4$$

Now that we named x, what number is it?

Welcome to College Algebra

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To answer this, we can start pick numbers from our list up there, and keep adding our 3 to them...

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If $x = 3$, then we caught: $3 + 3 = 6$

Welcome to College Algebra

Our numbers so far:

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Now that we named x, what number is it?
To answer this, we can start pick numbers from our list up there, and keep adding our 3 to them...
If $x = 0$, then we caught: $3 + 0 = 3$
That's not enough... Let's try something bigger.
If $x = 3$, then we caught: $3 + 3 = 6$
That's too much... Let's try something smaller.

Welcome to College Algebra

Our numbers so far:

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Now that we named x, what number is it?

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If $x = 0$, then we caught: $3 + 0 = 3$

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That's too much... Let's try something smaller.

If $x = 1$, then we caught: $3 + 1 = 4$

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If $x = 1$, then we caught: $3 + 1 = 4$

That's it! They need 1 antelope!

Welcome to College Algebra

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We will call this number that works our *solution*

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Although this technique is inefficient, it works!

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Don't worry, we will work on efficiency throughout the course!

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That's it! They need 1 antelope!
We will call this number that works our *solution*
Although this technique is inefficient, it works!
Don't worry, we will work on efficiency throughout the course!
For now, we are hunter/gatherers happy that we figured it out!

Welcome to College Algebra

Our numbers so far:

$$\{0, 1, 2, 3, ...\}$$

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As we gather more and hunt less, we have more time to think
In our time spent not hunting, we think about this Algebra

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What is the solution? What number for x works?

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As before, we can try numbers from our list above and check.

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Welcome to College Algebra

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We saw that 0 is important to addition, so we wonder:

$$3 + x = 0$$

What is the solution? What number for x works?
As before, we can try numbers from our list above and check.
If $x = 2$, then we caught: $3 + 2 = 5$
That's too much... Let's try something smaller.

Welcome to College Algebra

Our numbers so far:

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As before, we can try numbers from our list above and check.
If $x = 2$, then we caught: $3 + 2 = 5$
That's too much... Let's try something smaller.
If $x = 1$, then we caught: $3 + 1 = 4$

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If $x = 2$, then we caught: $3 + 2 = 5$
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If $x = 2$, then we caught: $3 + 2 = 5$
That's too much... Let's try something smaller.
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If $x = 2$, then we caught: $3 + 2 = 5$
That's too much... Let's try something smaller.
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That's too much... Let's try something smaller!
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That's too much... Let's try something smaller!!

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Our numbers so far:

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If $x = 2$, then we caught: $3 + 2 = 5$

That's too much... Let's try something smaller.

If $x = 1$, then we caught: $3 + 1 = 4$

That's too much... Let's try something smaller!

If $x = 0$, then we caught: $3 + 0 = 0$

That's too much... Let's try something smaller!!

We don't have any number smaller! Yet...

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That's too much... Let's try something smaller!
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That's too much... Let's try something smaller!!
We don't have any number smaller! Yet...
Just like when we starting including 0, we need new numbers!

Welcome to College Algebra

Our numbers so far:

$$\{0, 1, 2, 3, ...\}$$

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Our numbers so far:

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Just like when we starting including 0, we need new numbers!
None of our numbers solve our problem. So, we make one up!

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We call our new number that solves our equation: -3

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Just like when we starting including 0, we need new numbers!
None of our numbers solve our problem. So, we make one up!
We call our new number that solves our equation: -3
Even though we had to make it up, we have a solution:

$$3 + -3 = 0$$

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Our numbers so far:

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Just like when we starting including 0, we need new numbers!
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Even though we had to make it up, we have a solution:

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We don't want to stop with 3, though! Let's keep going.

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$$1 + -1 = 0$$

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$$1 + -1 = 0$$
$$2 + -2 = 0$$

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$$1 + -1 = 0$$

$$2 + -2 = 0$$

$$100 + -100 = 0$$

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We don't want to stop with 3, though! Let's keep going.

$$1 + -1 = 0$$

$$2 + -2 = 0$$

$$100 + -100 = 0$$

For *every* number in our list, let's do that same!

Welcome to College Algebra

Our numbers so far:

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Just like when we starting including 0, we need new numbers!
None of our numbers solve our problem. So, we make one up!
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Even though we had to make it up, we have a solution:

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We don't want to stop with 3, though! Let's keep going.

$$1 + -1 = 0$$

$$2 + -2 = 0$$

$$100 + -100 = 0$$

For *every* number in our list, let's do that same!
The negative of a number is the number we to add to it get 0.

Welcome to College Algebra

Our numbers so far:

$$\{0, 1, 2, 3, ...\}$$

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Just like when we starting including 0, we need new numbers!
None of our numbers solve our problem. So, we make one up!
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We don't want to stop with 3, though! Let's keep going.

$$1 + -1 = 0$$

$$2 + -2 = 0$$

$$100 + -100 = 0$$

For *every* number in our list, let's do that same!
The negative of a number is the number we to add to it get 0.
Our new number set is called the **Integers**, and labeled **Z**:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

Welcome to College Algebra

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We invented our new numbers so that for any number A:

$$A + -A = 0$$

Welcome to College Algebra

Our numbers so far:

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We invented our new numbers so that for any number A:

$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

Welcome to College Algebra

Our numbers so far:

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We invented our new numbers so that for any number A:

$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

Welcome to College Algebra

Our numbers so far:

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We invented our new numbers so that for any number A:

$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

But what is $- - 3$?

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

But what is $- - 3$?

The negative of a number is the number we to add to it get 0.

$- - 3$ is the number we add to -3 to get 0.

Welcome to College Algebra

Our numbers so far:

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$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

But what is $- - 3$?

The negative of a number is the number we to add to it get 0.

$- - 3$ is the number we add to -3 to get 0.

What number do we add to -3 to get 0?

Welcome to College Algebra

Our numbers so far:

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$$A + -A = 0$$

We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

But what is $--3$?

The negative of a number is the number we to add to it get 0.

$--3$ is the number we add to -3 to get 0.

What number do we add to -3 to get 0?

$$-3 + 3 = 0$$

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

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We first saw that -3 is the number we add to 3 to get 0

And that -2 is the number we add to 2 to get 0

And that -100 is the number we add to 100 to get 0...

But what is $--3$?

The negative of a number is the number we to add to it get 0.

$--3$ is the number we add to -3 to get 0.

What number do we add to -3 to get 0?

$$-3 + 3 = 0$$

Since 3 is the number we add to -3 to get 0, we have:

$$--3 = 3$$

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We first saw that -3 is the number we add to 3 to get 0

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And that -100 is the number we add to 100 to get 0...

But what is $- - 3$?

The negative of a number is the number we to add to it get 0.

$- - 3$ is the number we add to -3 to get 0.

What number do we add to -3 to get 0?

$$-3 + 3 = 0$$

Since 3 is the number we add to -3 to get 0, we have:

$$- - 3 = 3$$

Again, there's nothing special about 3 here, for any number A:

$$- - A = A$$

Welcome to College Algebra

Our numbers so far:

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We invented our new numbers so that for any number A:

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Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

$$A + -A = 0$$

Also, remember our property of Subtraction:

$$A - A = 0$$

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

$$A + -A = 0$$

Also, remember our property of Subtraction:

$$A - A = 0$$

These are nearly identical!

$$A + -A = A - A$$

Welcome to College Algebra

Our numbers so far:

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We invented our new numbers so that for any number A:

$$A + -A = 0$$

Also, remember our property of Subtraction:

$$A - A = 0$$

These are nearly identical!

$$A + -A = A - A$$

And it doesn't matter if we start with A or another number B:

$$B + -A = B - A$$

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

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Also, remember our property of Subtraction:

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And it doesn't matter if we start with A or another number B:

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So, we can replace subtraction, with addition of a negative!

Welcome to College Algebra

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These are nearly identical!

$$A + -A = A - A$$

And it doesn't matter if we start with A or another number B:

$$B + -A = B - A$$

So, we can replace subtraction, with addition of a negative!

And we can replace addition of a negative with subtraction

Welcome to College Algebra

Our numbers so far:

$$\mathbf{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

We invented our new numbers so that for any number A:

$$A + -A = 0$$

Also, remember our property of Subtraction:

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These are nearly identical!

$$A + -A = A - A$$

And it doesn't matter if we start with A or another number B:

$$B + -A = B - A$$

So, we can replace subtraction, with addition of a negative!

And we can replace addition of a negative with subtraction

This versatility will be useful!

Welcome to College Algebra

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We want to add these numbers

$$3 + 2 + 4 = ?$$

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$$3 + 2 + 4 = ?$$

But with addition we can only add 2 at a time.

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$$3 + 2 + 4 = ?$$

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Which 2 do we add first?

If we add 3 and 2 first, we get:

$$(3 + 2) + 4 =$$

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$$3 + 2 + 4 = ?$$

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Which 2 do we add first?

If we add 3 and 2 first, we get:

$$(3 + 2) + 4 = (5) + 4$$

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$$3 + 2 + 4 = ?$$

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Which 2 do we add first?

If we add 3 and 2 first, we get:

$$(3 + 2) + 4 = (5) + 4 = 9$$

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$$3 + (2 + 4) = 3 + (6)$$

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It doesn't matter what order we add the three numbers!

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It doesn't matter what order we add the three numbers!

Associate Property of Addition: For A, B, *and* C

$$(A + B) + C = A + (B + C)$$

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Example 5: In the next hunting trip, you catch 3 and your partners catch 3 and 3

So, in total you caught:

$$3 + 3 + 3 =$$

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As our society expands, we need more food, so we need 4 hunters, who each catch 3, for a total of:

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$$\underbrace{\underbrace{3+3}_{6}+3}_{9} = 9$$

As our society expands, we need more food, so we need 4 hunters, who each catch 3, for a total of:

$$\underbrace{\underbrace{\underbrace{3+3}_{6}+3}_{9}+3}_{12} = 12$$

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$$3 + 3 + 3 = 9$$

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Continuing this, what if we have 5 hunters who each catch 3?

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$$\underbrace{\underbrace{\underbrace{\underbrace{3 + 3}_{6} + 3}_{9} + 3}_{12} + 3}_{15} = 15$$

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Now, what if we have 10 hunters who each catch 3 antelope?

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Now, what if we have 10 hunters who each catch 3 antelope?

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 =$$

This Addition problem isn't difficult, but it's *tedious*.

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Multiplication $A \cdot B$ means to add A to itself B times.

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Our problem becomes:

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$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 3 \cdot 10$$

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Continuing this, what if we have 5 hunters who each catch 3?

$$\underbrace{\underbrace{\underbrace{\underbrace{3 + 3}_{6} + 3}_{9} + 3}_{12} + 3}_{15} = 15$$

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$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 3 \cdot 10 = 30$$

Welcome to College Algebra

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We can visualize multiplication in terms of blocks

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Example: $4 \cdot 3$

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By definition: $4 \cdot 3 = \underbrace{4 + 4 + 4}_{3 \textit{ times}} = 12$

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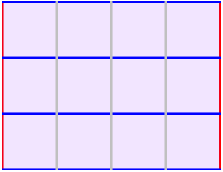
By definition: $4 \cdot 3 = \underbrace{4 + 4 + 4}_{3\ \textit{times}} = 12$

So we can visualize multiplication of $4 \cdot 3$ as a rectangle of width 4 and height 3



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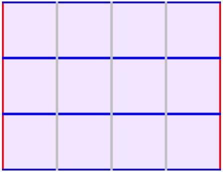

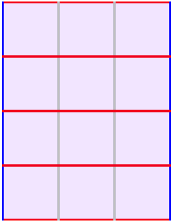

Note: This helps us see that if we compute $3 \cdot 4$ we get the same rectangle (but on its side)

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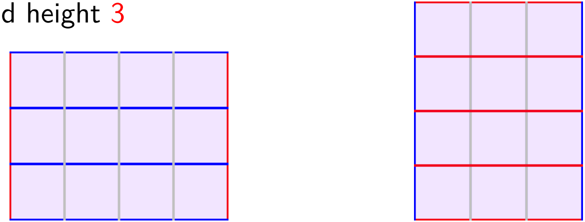

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So we see: $4 \cdot 3 = 3 \cdot 4$

Welcome to College Algebra

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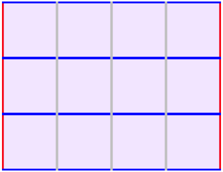

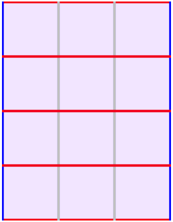

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Commutative Property of Multiplication: For any A, B

$$A \cdot B = B \cdot A$$

Welcome to College Algebra

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We saw both Addition and Multiplication are Commutative:

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$A + B = B + A$ $\qquad$ $A \cdot B = B \cdot A$

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Similar to Addition; Multiplication is also Associative.

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Similar to Addition; Multiplication is also Associative.

For A, B, *and* C

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

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For A, B, $and\, C$

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So, if we are only multiplying, we can multiply in any order

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Example: $2 \cdot x \cdot 5$

Welcome to College Algebra

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For A, B, $and\, C$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

So, if we are only multiplying, we can multiply in any order

Example: $2 \cdot x \cdot 5 = 2 \cdot 5 \cdot x$

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$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

So, if we are only multiplying, we can multiply in any order

Example: $2 \cdot x \cdot 5 = 2 \cdot 5 \cdot x = 10 \cdot x$

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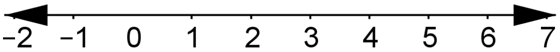

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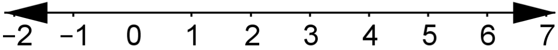

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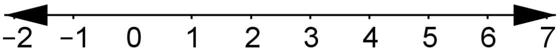

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But later in the semester we will add to it 1 more time!

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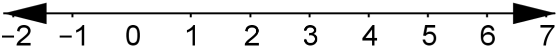

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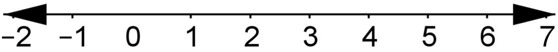

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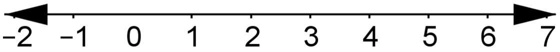

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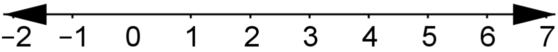

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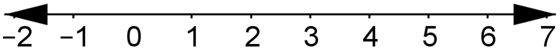

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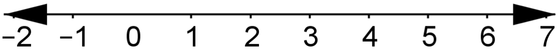

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Example 2: $\{0, \pi, 10\}$

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We can understand Subtraction as Addition of negatives

Example: $7 - 4 = 7 + (-4) = 3$

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Commutative Property of Addition: For any numbers A, B

$$A + B = B + A$$

Associate Property of Addition: For A, B, $and\,C$

$$(A + B) + C = A + (B + C)$$

0 is the Additive Identity: For any number A

$$A + 0 = A = 0 + A$$

Subtraction:

Example: $7 - 4 = 3$

We can understand Subtraction as Addition of negatives

Example: $7 - 4 = 7 + (-4) = 3$

This allows us to use all the properties of Addition, which we don't have for Subtraction

Welcome to College Algebra

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We have seen the 4 basic operations to combine numbers

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Subtraction

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Multiplication:

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Multiplication:

Example: $4 \cdot 3 = 12$

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Example: $4 \cdot 3 = 12$

Commutative Property of Multiplication: For any A, B

$$A \cdot B = B \cdot A$$

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We can understand Division as Multiplication of reciprocals

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Example: $12 \div 4 = 12 \cdot \frac{1}{4} = 3$

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In general, we can change division into multiplication by the reciprocal which allows us to use all the properties of Multiplication, which we don't have for Division

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Example: $24 \div \frac{3}{5} = 24 \cdot \frac{5}{3} = 40$