

Solving a Rational Inequality

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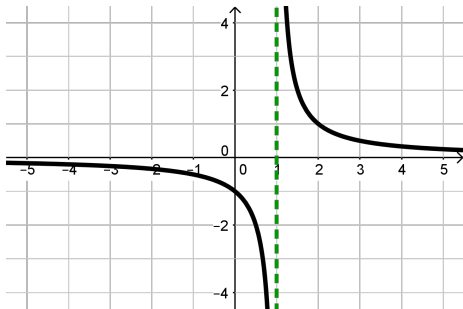
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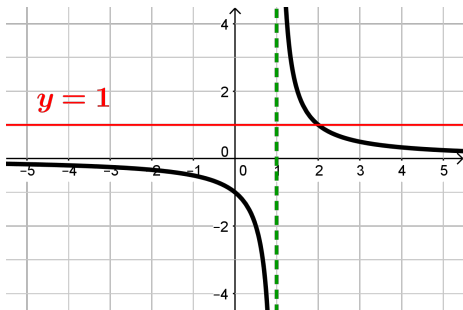
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Now we want to know when this function $y = f(x) \leq 1$



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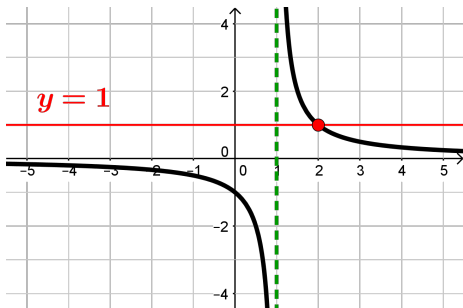
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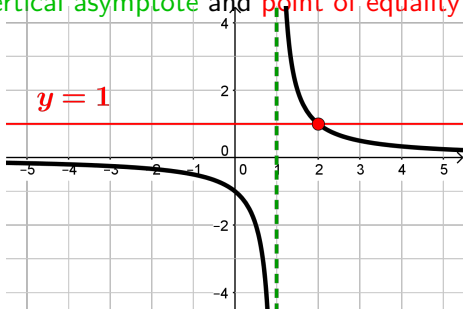
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We need to find x values at the vertical asymptote and point of equality



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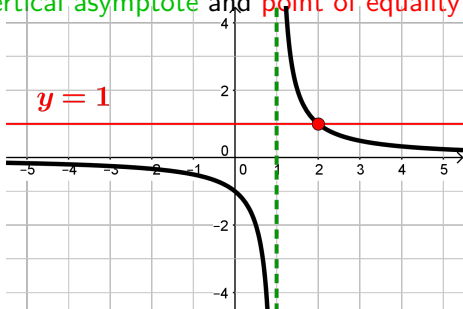
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Vertical Asymptote



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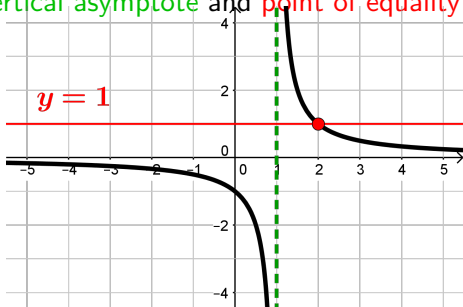
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Vertical Asymptote

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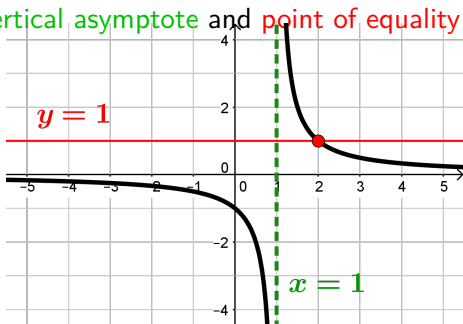
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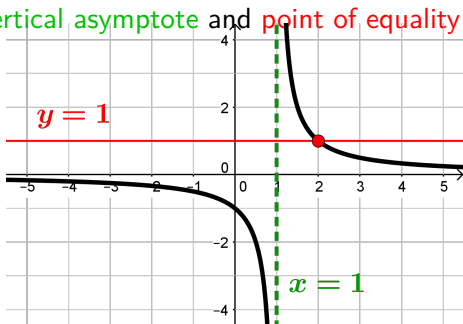
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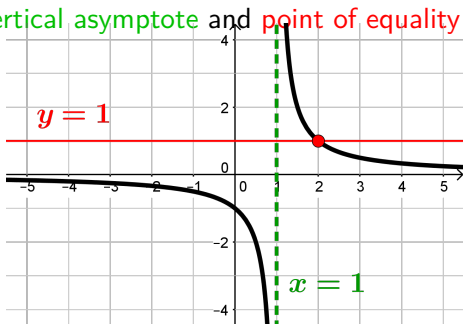
Vertical Asymptote

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▶ Solving $\frac{1}{x-1} = 1$

We find: $x = 2$



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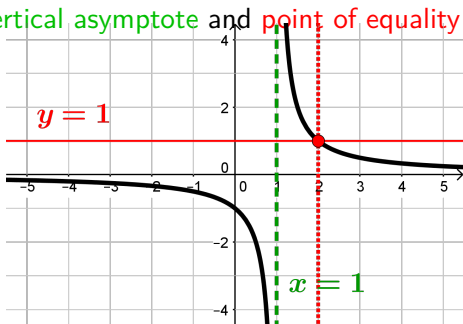
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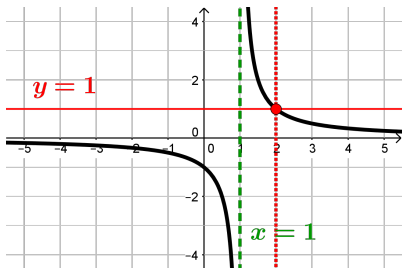
We find: $x = 2$



Now we can break our x number line (axis) into 3 regions

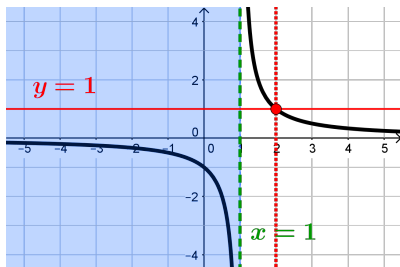
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Solving a Rational Inequality

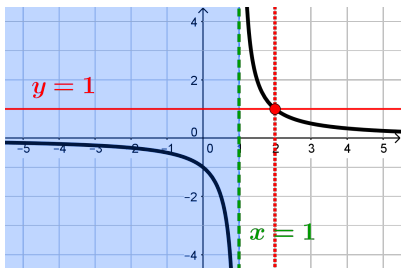
Solve the Rational Inequality: $\frac{1}{x-1} \leq 1$



Checking the region $x < 1$ we can let $x = 0$

Solving a Rational Inequality

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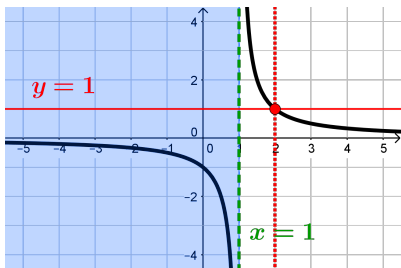


Checking the region $x < 1$ we can let $x = 0$

For $x = 0$: $f(0) = \frac{1}{0-1}$

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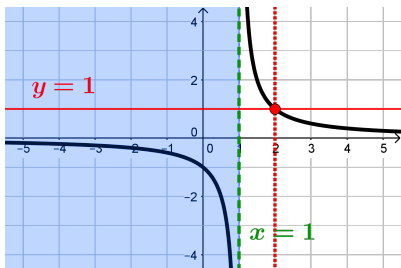


Checking the region $x < 1$ we can let $x = 0$

$$\text{For } x = 0: f(0) = \frac{1}{0-1} = -1$$

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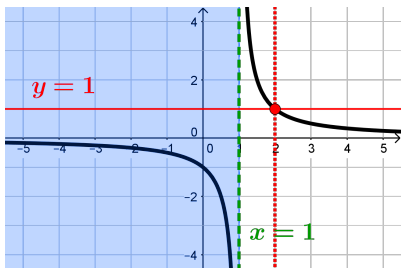


Checking the region $x < 1$ we can let $x = 0$

For $x = 0$: $f(0) = \frac{1}{0-1} = -1 < 1$

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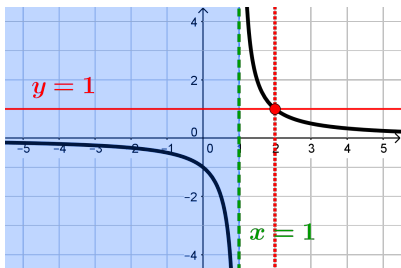
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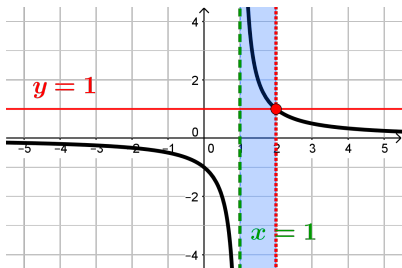
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So, $x = 0$ is a solution $\rightarrow x$ is a solution for all $x < 1$

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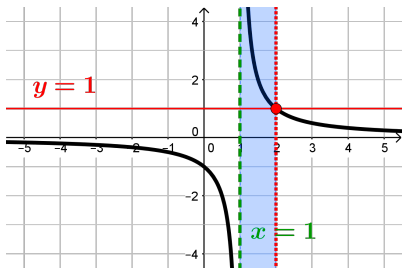
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So, $x = 0$ is a solution $\rightarrow x$ is a solution for all $x < 1$

Checking the region $1 < x < 2$ we can let $x = 1.5$

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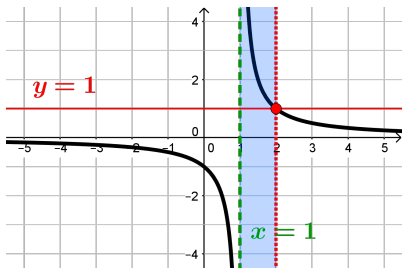
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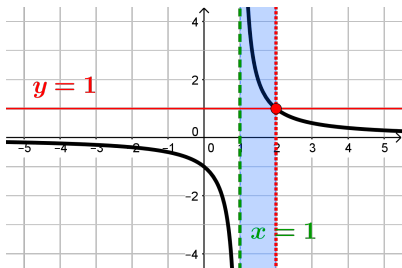
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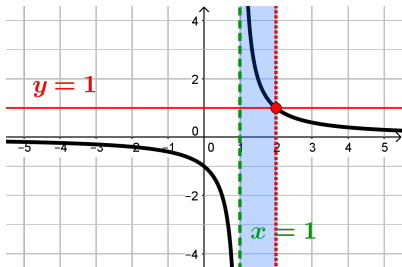
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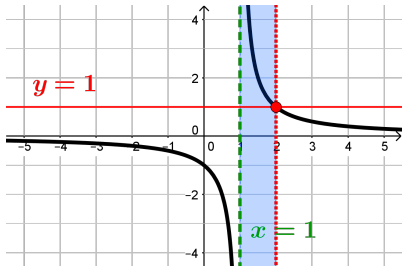
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So, $x = 1.5$ is not a solution

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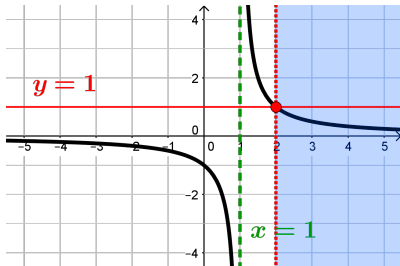
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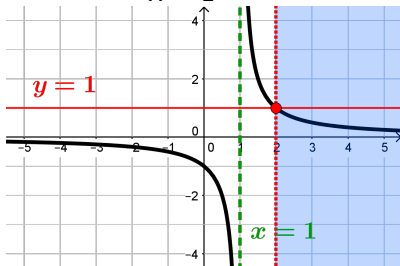
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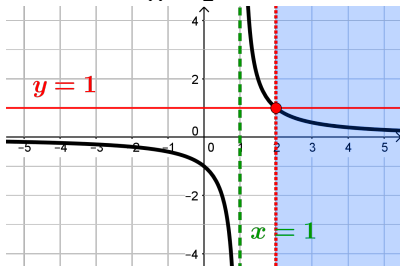
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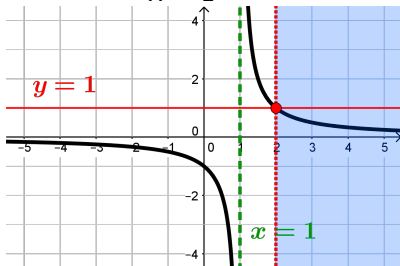
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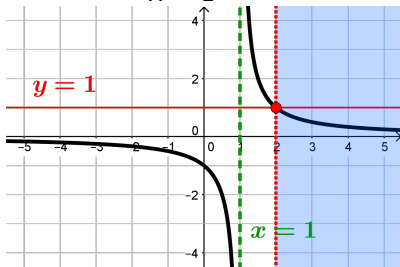
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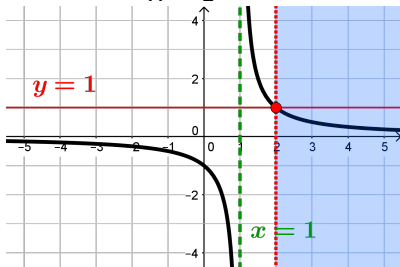
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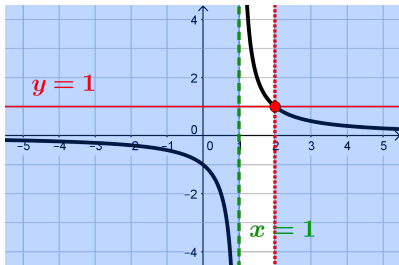
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Conclusion: The solutions to $\frac{1}{x-1} \leq 1$ are: $(-\infty, 1) \cup [2, \infty)$