

## Graphing Rational Functions - General

# Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

▶ We saw that if  $D(a) = 0$  and  $P(a) \neq 0$  then  $f(x)$  has a vertical asymptote at  $x = a$

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

▶ We saw that if  $D(a) = 0$  and  $P(a) \neq 0$  then  $f(x)$  has a vertical asymptote at  $x = a$

Furthermore, we saw that  $f(x) \rightarrow \pm\infty$  as  $x$  gets near  $a$

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

▶ We saw that if  $D(a) = 0$  and  $P(a) \neq 0$  then  $f(x)$  has a vertical asymptote at  $x = a$

Furthermore, we saw that  $f(x) \rightarrow \pm\infty$  as  $x$  gets near  $a$

Like, with all of our functions so far, the  $x$ - and  $y$ -intercepts will play important roles in our graph.

## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

▶ We saw that if  $D(a) = 0$  and  $P(a) \neq 0$  then  $f(x)$  has a vertical asymptote at  $x = a$

Furthermore, we saw that  $f(x) \rightarrow \pm\infty$  as  $x$  gets near  $a$

Like, with all of our functions so far, the  $x$ - and  $y$ -intercepts will play important roles in our graph.

As with polynomials, we will always want to understand the End Behavior of our graph.



## Graphing Rational Functions - General

▶ We have seen how to sketch the basic graph  $f(x) = \frac{1}{x}$

▶ Using Graph Shifting we sketched the graph of  $f(x) = \frac{1}{x-1}$

How do we sketch the graph of a Rational Function in general?

$$f(x) = \frac{P(x)}{D(x)}$$

▶ We saw that if  $D(a) = 0$  and  $P(a) \neq 0$  then  $f(x)$  has a vertical asymptote at  $x = a$

Furthermore, we saw that  $f(x) \rightarrow \pm\infty$  as  $x$  gets near  $a$

Like, with all of our functions so far, the  $x$ - and  $y$ -intercepts will play important roles in our graph.

As with polynomials, we will always want to understand the End Behavior of our graph.

Recall: The End Behavior is what happens to  $f(x)$  as  $x \rightarrow \pm\infty$  goes off the left and right hand sides of the graph.

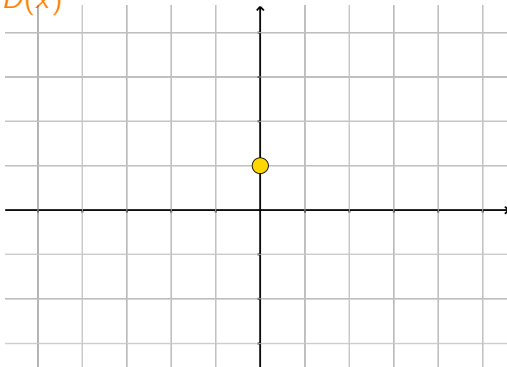
## Graphing Rational Functions - General

## Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:



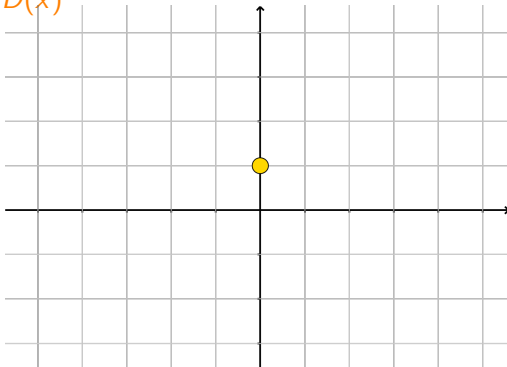
## Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int



## Graphing Rational Functions - General

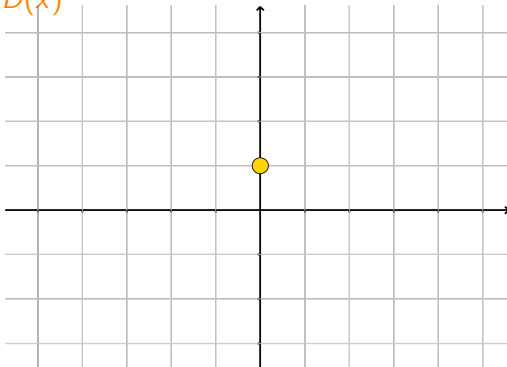
To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int

The  $x$ -int



## Graphing Rational Functions - General

To graph a Rational Function:

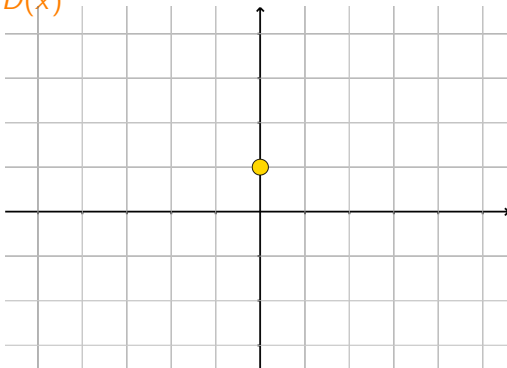
$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int

The  $x$ -int

Vertical asymptotes



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

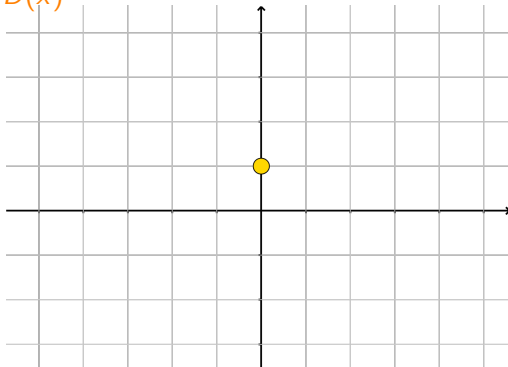
We need to find:

The  $y$ -int

The  $x$ -int

Vertical asymptotes

The End Behavior



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

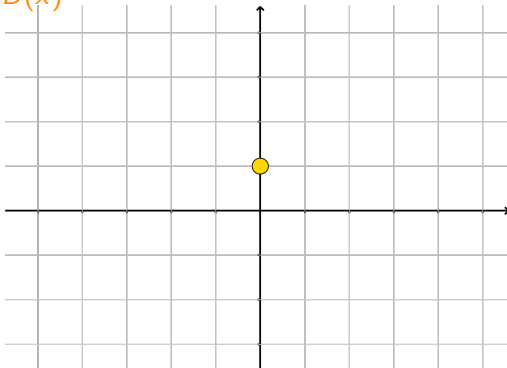
We need to find:

The  $y$ -int:  $x = 0$

The  $x$ -int

Vertical asymptotes

The End Behavior





# Graphing Rational Functions - General

To graph a Rational Function:

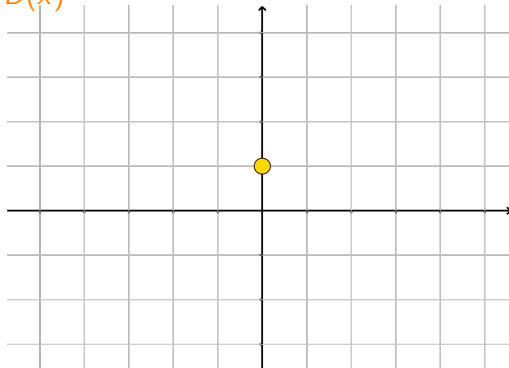
$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

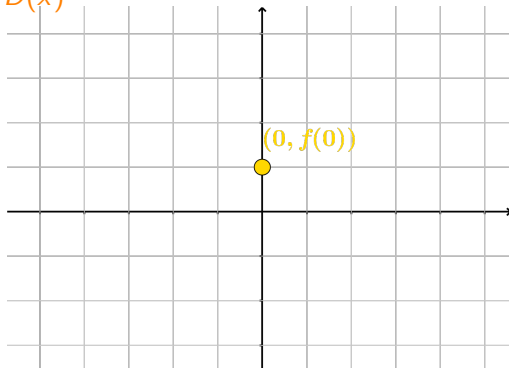
$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

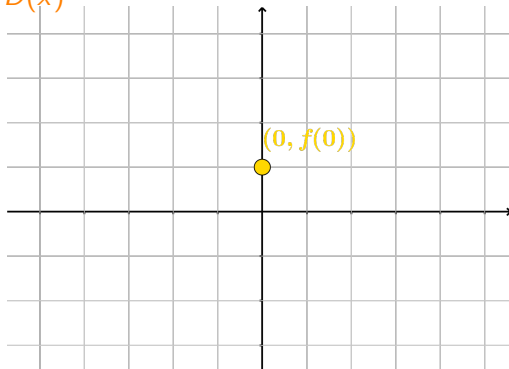
$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The **y-int**:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The **x-int**:  $y = f(x) = 0$



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

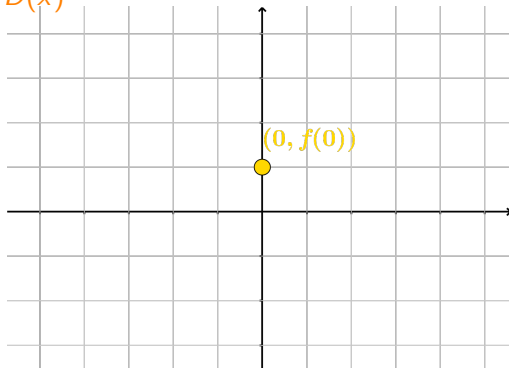
We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

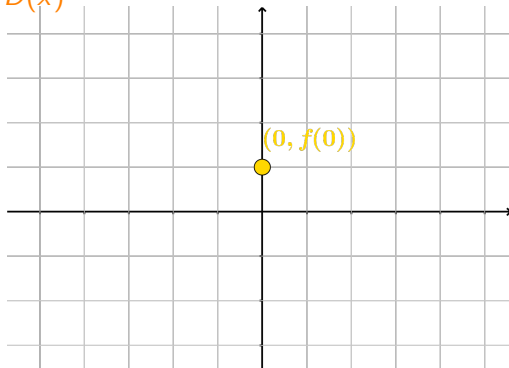
The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

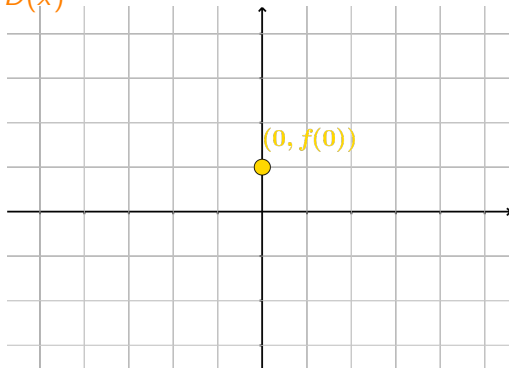
$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$



Vertical asymptotes

The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

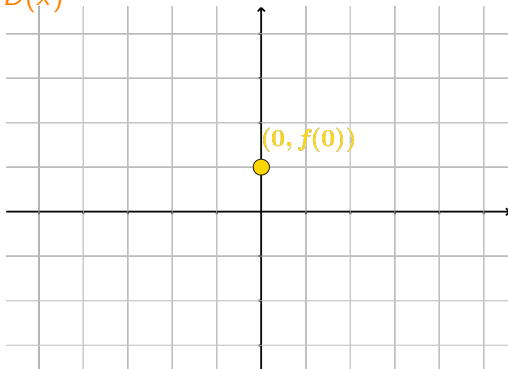
We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes

The End Behavior



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

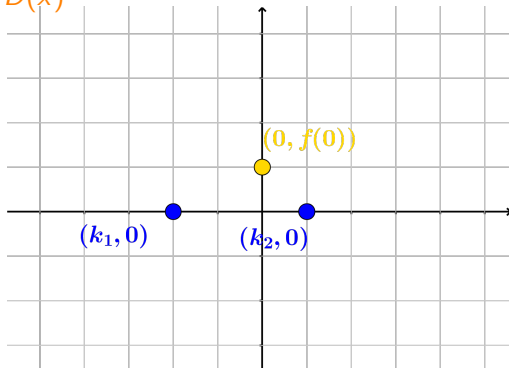
We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes

The End Behavior





# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

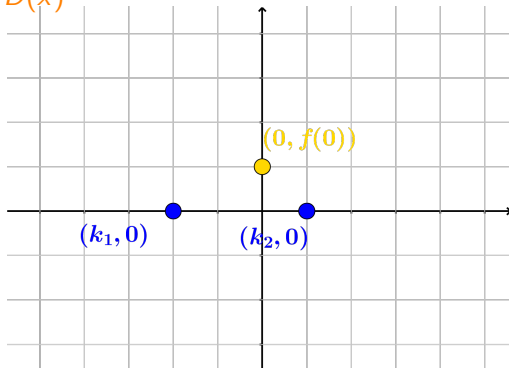
We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$



The End Behavior

# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

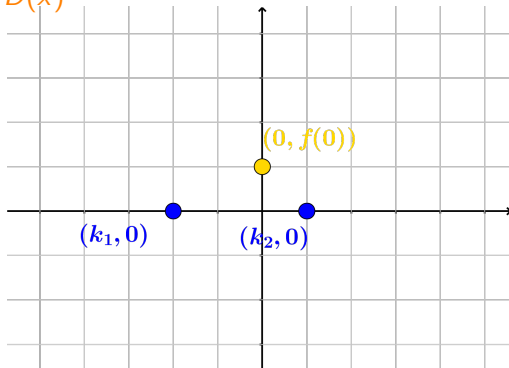
This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

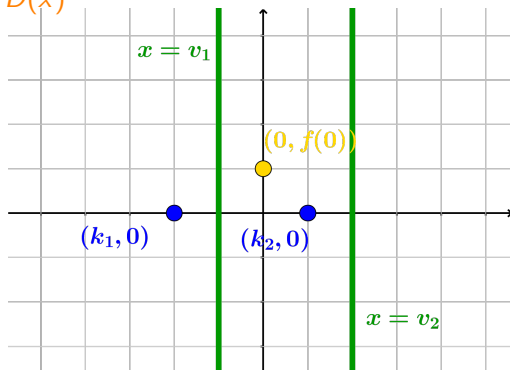
This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

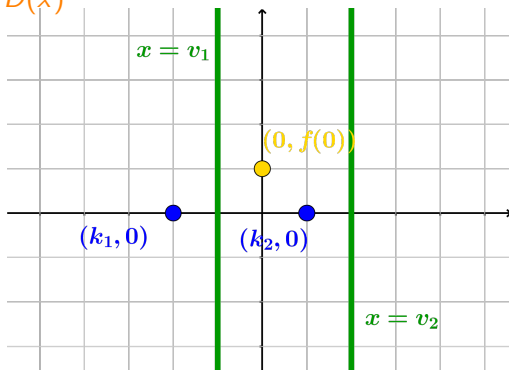
This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

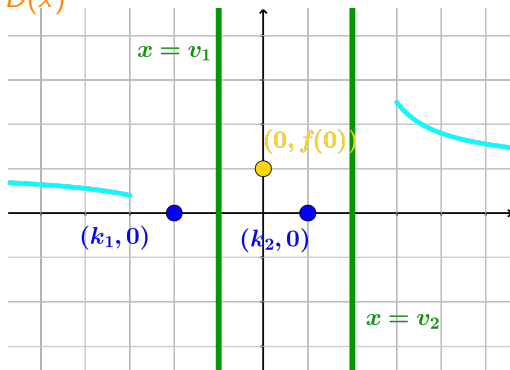
This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

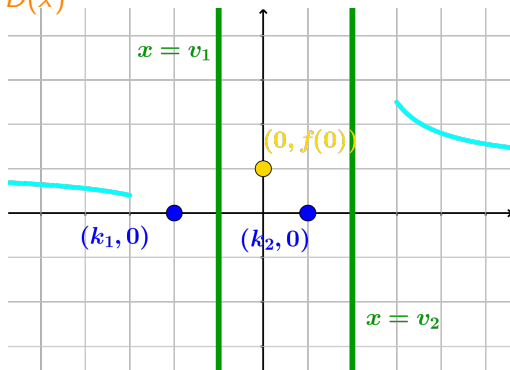
$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$

[▶ You can see](#) a more extensive lecture of End Behavior for Rationals here



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

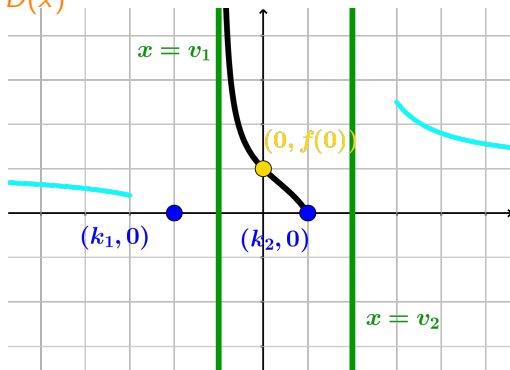
$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$

▶ You can see a more extensive lecture of End Behavior for Rationals here



# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

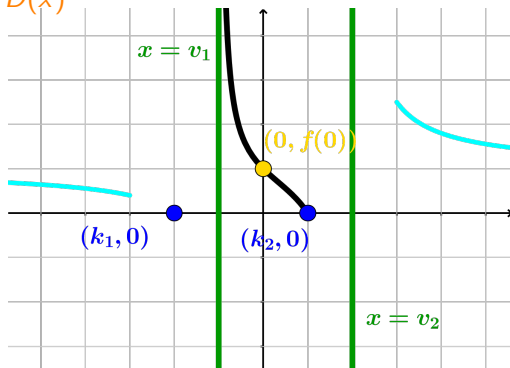
Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$

▶ You can see a more extensive lecture of End Behavior for Rationals here

▶ Like Polynomials we need to check if  $f(x) > 0$  or  $f(x) < 0$  on some intervals





# Graphing Rational Functions - General

To graph a Rational Function:

$$f(x) = \frac{P(x)}{D(x)}$$

We need to find:

The  $y$ -int:  $x = 0$

$$y = f(0) \rightarrow (0, f(0))$$

The  $x$ -int:  $y = f(x) = 0$

We need to solve  $0 = \frac{P(x)}{D(x)}$

We can by solving:  $0 = P(x)$

This gives:  $x = k_1, k_2, \text{etc} \dots$

$(k_1, 0), (k_2, 0), \text{etc} \dots$

Vertical asymptotes:  $D(x) = 0$

Solving this polynomial gives:  $x = v_1, x = v_2, \text{etc} \dots$

The End Behavior:  $x \rightarrow \pm\infty$

▶ You can see a more extensive lecture of End Behavior for Rationals here

▶ Like Polynomials we need to check if  $f(x) > 0$  or  $f(x) < 0$  on some intervals

Since there are no more  $x$ -int we know where  $f(x)$  can change sign

