

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

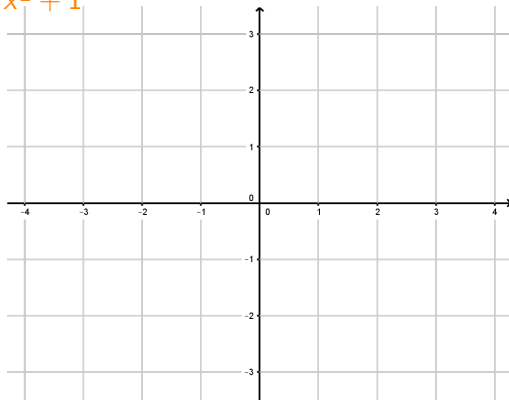
$$f(x) = \frac{1}{x^2 + 1}$$

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:



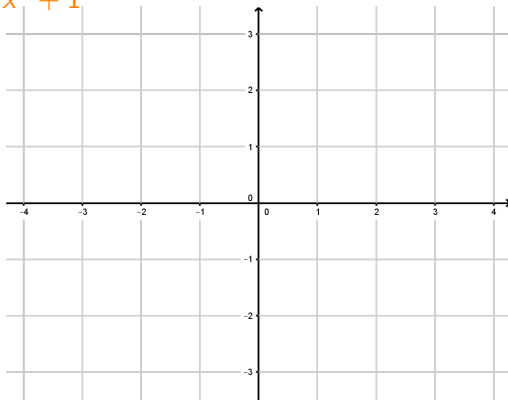
Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int



Graphing Rational Functions - Example 4

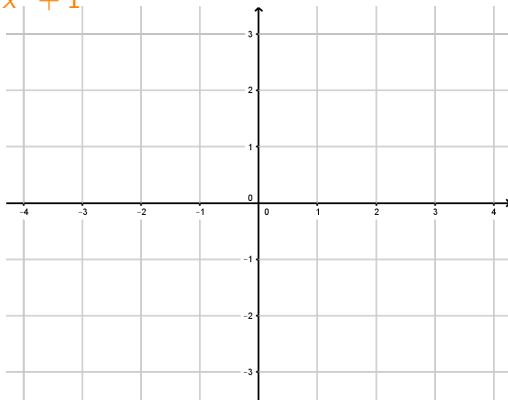
Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int

The x -int



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

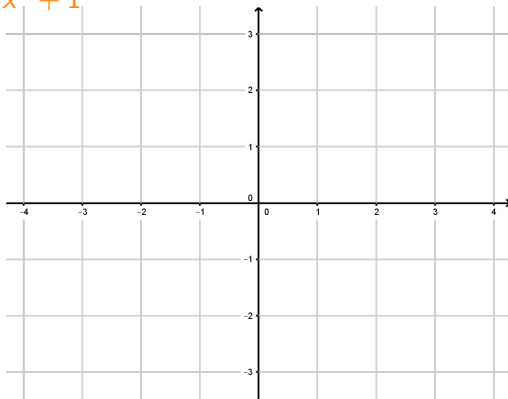
$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int

The x -int

Vertical asymptotes



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

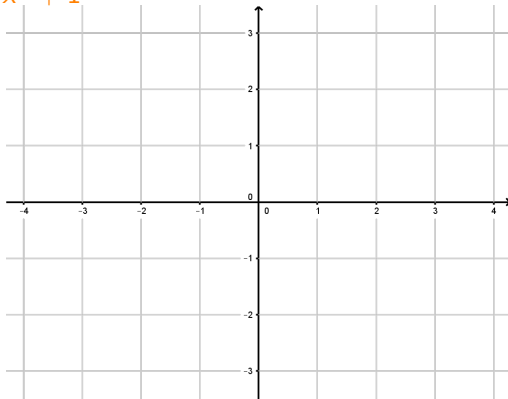
We need to find:

The y -int

The x -int

Vertical asymptotes

The End Behavior



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

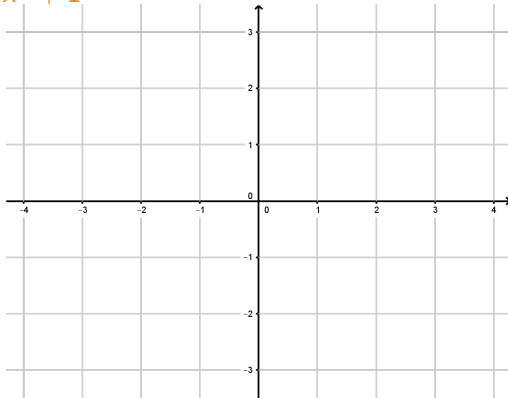
We need to find:

The y -int: $x = 0$

The x -int

Vertical asymptotes

The End Behavior



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

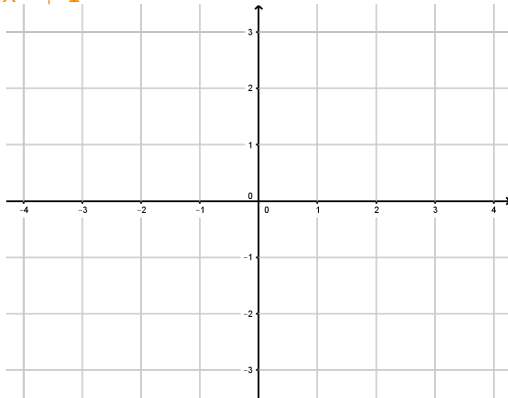
$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int



Vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

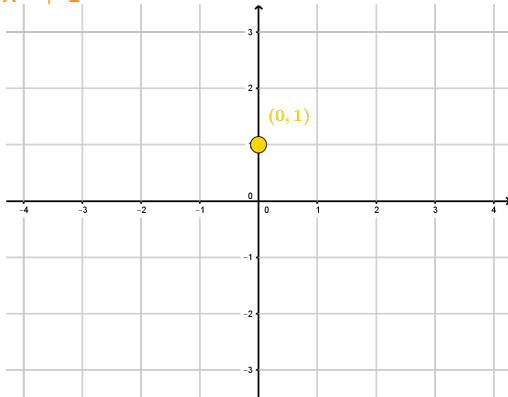
$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int



Vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

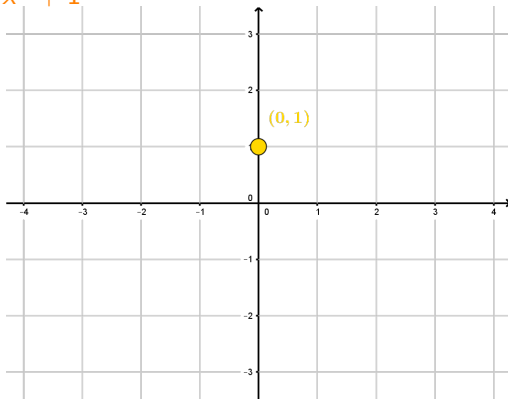
$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$



Vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

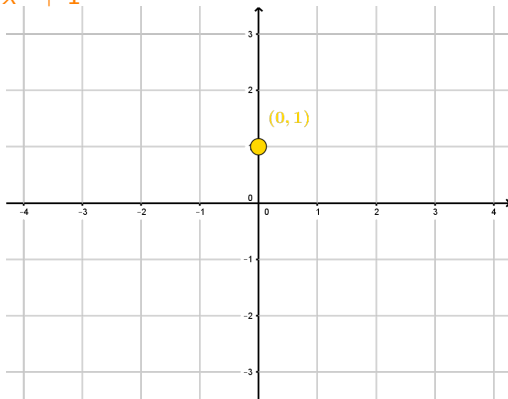
We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$



Vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

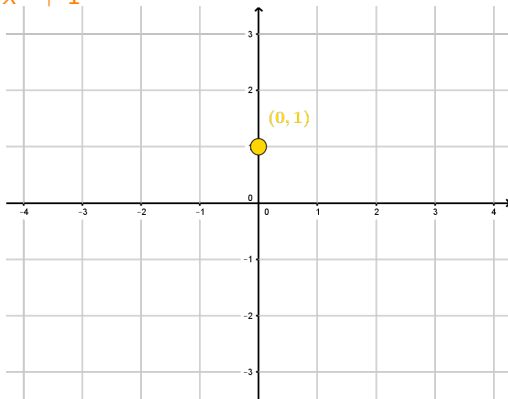
The x -int: $y = f(x) = 0$

We need to solve $0 = \frac{1}{x^2 + 1}$

By solving $0 = 1$

Vertical asymptotes

The End Behavior



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

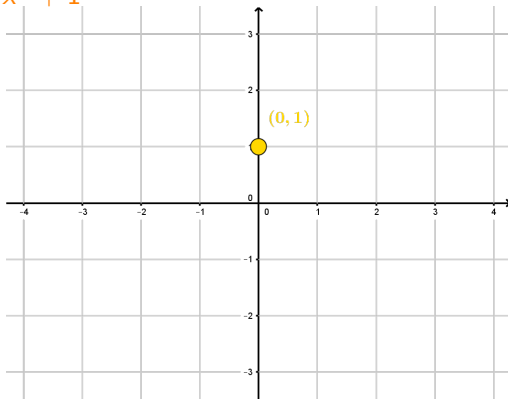
The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes



The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

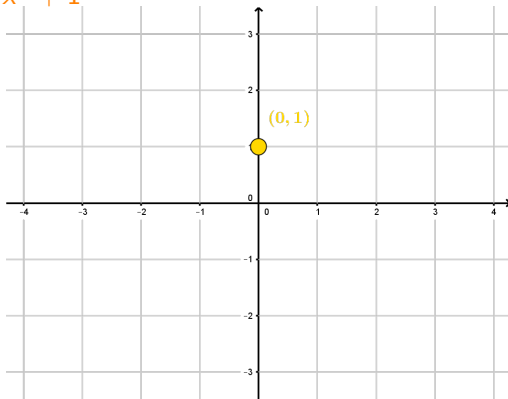
The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes



The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

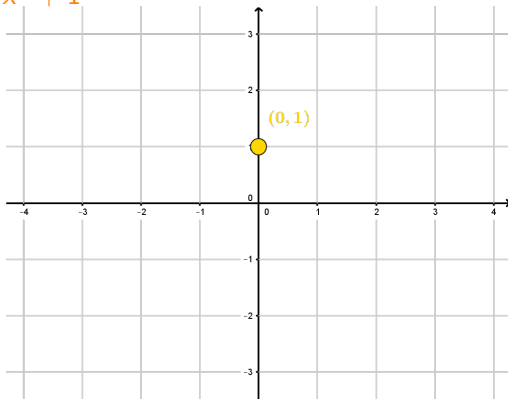
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

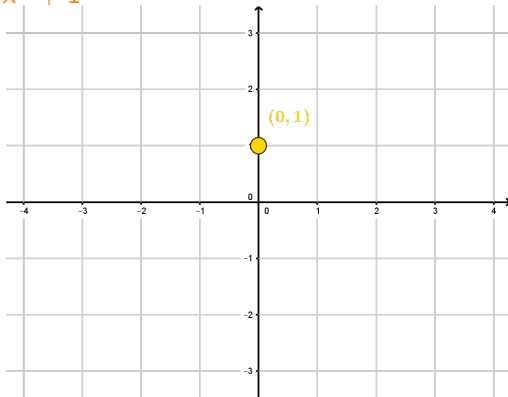
This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$

This also has no real solutions (just $\pm i$)

The **End Behavior**



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

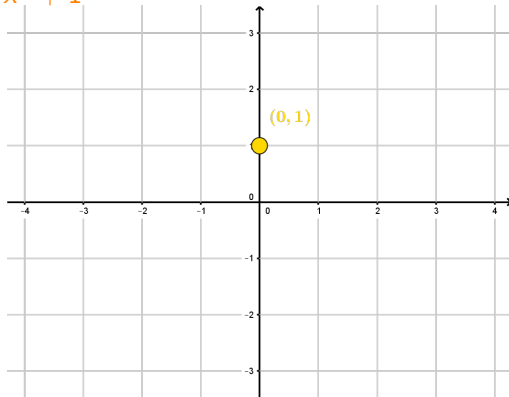
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

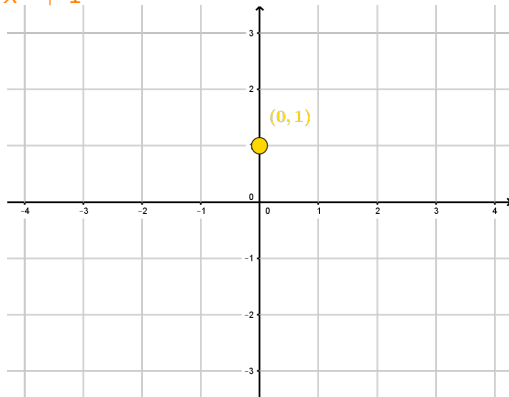
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

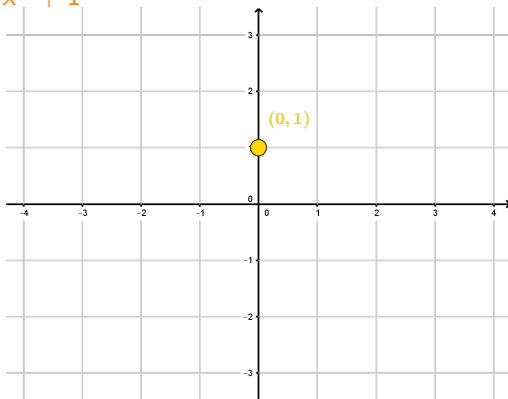
This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$

This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior: $x \rightarrow \pm\infty$



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

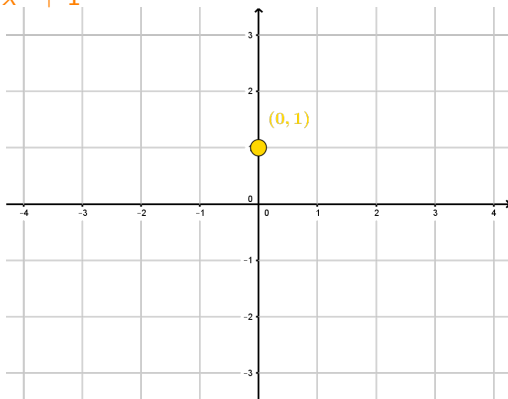
This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$

This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior: $x \rightarrow \pm\infty$



Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

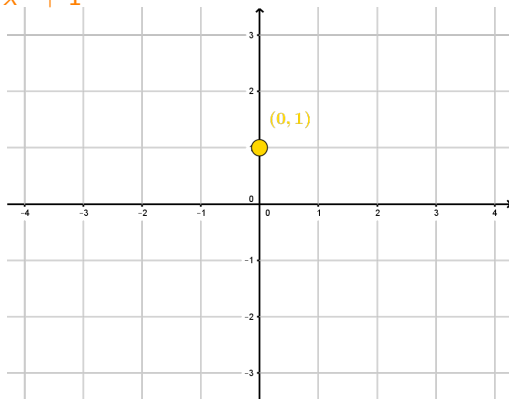
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): **No vertical asymptotes**

The **End Behavior**: $x \rightarrow \pm\infty$

► We saw that $\frac{1}{x^2 + 1} \approx 0$ since $\deg(1) < \deg(x^2 + 1)$

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

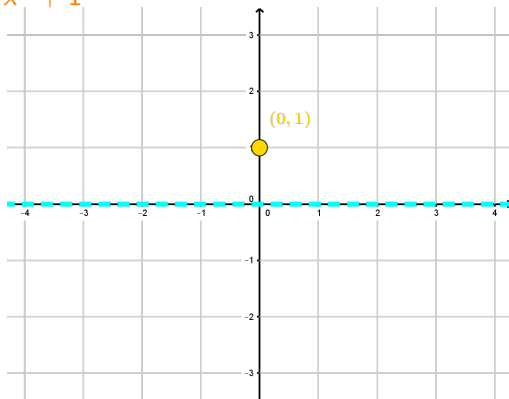
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior: $x \rightarrow \pm\infty$

► We saw that $\frac{1}{x^2 + 1} \approx 0$ since $\deg(1) < \deg(x^2 + 1)$

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

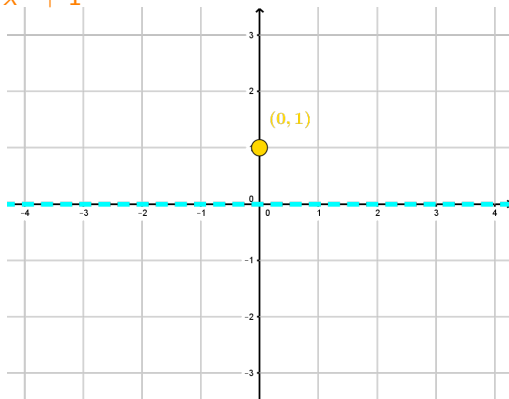
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior: $x \rightarrow \pm\infty$

► We saw that $\frac{1}{x^2 + 1} \approx 0$ since $\deg(1) < \deg(x^2 + 1)$

Since there are no more x -int we know $f(x)$ cannot change sign

Graphing Rational Functions - Example 4

Example: Sketch the graph of:

$$f(x) = \frac{1}{x^2 + 1}$$

We need to find:

The y -int: $x = 0$

$$f(0) = \frac{1}{0^2 + 1} = 1 \rightarrow (0, 1)$$

The x -int: $y = f(x) = 0$

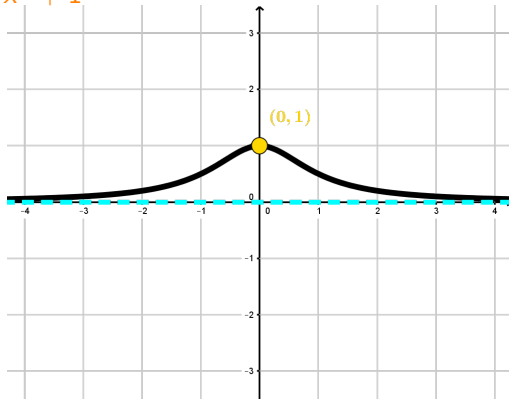
$$\text{We need to solve } 0 = \frac{1}{x^2 + 1}$$

By solving $0 = 1$

This means no x -int

Vertical asymptotes:

$$x^2 + 1 = 0$$



This also has no real solutions (just $\pm i$): No vertical asymptotes

The End Behavior: $x \rightarrow \pm\infty$

► We saw that $\frac{1}{x^2 + 1} \approx 0$ since $\deg(1) < \deg(x^2 + 1)$

Since there are no more x -int we know $f(x)$ cannot change sign