

Graphing Rational Functions - End Behavior General

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Let's look at this in 3 cases:

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2. If $m = n$ then $f(x) \approx \frac{a_n x^n}{b_m x^n} \approx \frac{a_n}{b_m}$
3. If $m < n$ then polynomial division gives: $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$

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For $x \rightarrow \pm\infty$: $\frac{R(x)}{D(x)} \approx 0$ from case 1

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For $x \rightarrow \pm\infty$: $\frac{R(x)}{D(x)} \approx 0$ from case 1

For $x \rightarrow \pm\infty$: $\frac{P(x)}{D(x)} \approx Q(x)$