Finding Roots of Polynomials - Example 3 - Recap Example: Find the roots of  $P(x) = x^5 + x^4 - 5x^3 - x^2 + 8x - 4$ 

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#### Conclusion:

The roots of P(x) are 1 of multiplicity 3 and -2 of multiplicity 2