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How do we solve $(x^3 - 3x + 2) = 0$?

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$x^5 + x^4 - 5x^3 - x^2 + 8x - 4 = 0$

Possible values of roots $\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}$

We need to evaluate $P(x)$ at our possible roots, to see which are roots.

$P(1) = 0$ $\quad P(2) = 16 \neq 0$ $\quad P(4) = 972 \neq 0$

$P(-1) = -8 \neq 0$ $\quad P(-2) = 0$ $\quad P(-4) = -500 \neq 0$

So, the only solutions we found are: $x = 1, -2$

We should have 5 solutions, what are the other solutions?

Since $x = 1, -2$ are roots, ▸ The Factoring Theorem tells us:

$$P(x) = x^5 + x^4 - 5x^3 - x^2 + 8x - 4 = (x-1)(x+2)Q(x)$$

To find $Q(x)$ we ▸ need to compute $\frac{x^5+x^4-5x^3-x^2+8x-4}{(x-1)(x+2)}$: $Q(x) = x^3 - 3x + 2$

So, $0 = x^5 + x^4 - 5x^3 - x^2 + 8x - 4 = (x-1)(x+2)(x^3 - 3x + 2)$

▸ By the Zero-Product Property: $(x-1) = 0$ OR $(x+2) = 0$ OR $(x^3 - 3x + 2) = 0$

The first two equations lead to the roots we already found: $x = 1, -2$

How do we solve $(x^3 - 3x + 2) = 0$?

Like the original equation, we use the ▸ Rational Root Theorem

Finding Roots of Polynomials - Example 3

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So, the roots of $P(x)$ are 1 of multiplicity 3 and -2 of multiplicity 2