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Conc: The roots of $P(x) = 3x^3 + 2x^2 - 7x + 2$ are: $x = 1, \sqrt{2}, -\sqrt{2}$

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So, the only solution we found was: $x = 1$

What are the other solutions?

Since $x = 1$ is a root, ▸ The Factoring Theorem tells us:

$$P(x) = x^3 - x^2 - 2x + 2 = (x - 1)Q(x)$$

To find $Q(x)$ we ▸ need to compute $\frac{x^3-x^2-2x+2}{(x-1)}$: $Q(x) = x^2 - 2$

So, $0 = x^3 - x^2 - 2x + 2 = (x - 1)(x^2 - 2)$

▸ By the Zero-Product Property

$(x - 1) = 0$ $\qquad$ $(x^2 - 2) = 0$

$x = 1$ is the root we knew $\qquad$ ▸ We can solve this to find: $x = \pm\sqrt{2}$

Conc: The roots of $P(x) = 3x^3 + 2x^2 - 7x + 2$ are: $x = 1, \sqrt{2}, -\sqrt{2}$

Note: The RRT did not find the roots $\pm\sqrt{2}$ since they are not rational.