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then p divides a_0 and q divides a_n

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We need to evaluate $P(x)$ at our possible roots, to see which are roots.

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$$P(1)$$

$$P(-1)$$

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► The FTA tells us that $P(x)$ has 3 roots because $\deg(P) = 3$

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Conclusion: The roots of $P(x) = 3x^3 + 2x^2 - 7x + 2$ are: $x = -2, 1, \frac{1}{3}$