

# Inverse Functions - Intro

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Since a function can't give both outputs; this inverse is not function!

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