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Example:

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Furthermore, a number that when multiplied by nm produces 1 is: $\frac{1}{mn}$

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Furthermore, a number that when multiplied by nm produces 1 is: $\frac{1}{mn}$
Conclusion: $\frac{1}{m} \cdot \frac{1}{n} = \frac{1}{mn}$

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We found that:

$$\frac{1}{m} \cdot \frac{1}{n} = \frac{1}{mn} \qquad \text{and} \qquad m \cdot \frac{1}{n} = \frac{m}{n}$$

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We can rearrange this because multiplication is commutative.

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In other words, to multiply two fractions, we multiply across the top and across the bottom

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