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In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$

Note: Our definition only makes sense if m is a positive integer

Now we know what happens with Multiplication; what about Division?

$$\frac{x^5}{x^3} = \frac{\overbrace{x \cdot x \cdot x \cdot x \cdot x}^{5-times}}{\underbrace{x \cdot x \cdot x}_{3-times}} = x^2$$

In General: For any numbers m and n:

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The only way that $x^n = 0$ is if $x = 0$

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Note: Since we are dividing by x^n, we need to make sure that

$$x^n \neq 0$$

The only way that $x^n = 0$ is if $x = 0$

So, this property only holds if $x \neq 0$

Power Rules of Exponents

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But this assumes that $m > n$

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Let's look at when $m = n$; i.e. top and bottom have the same exponent

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$$\frac{x^m}{x^m} =$$

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If the power rule holds:

$$1 = \frac{\cancel{x^m}}{\cancel{x^m}} = x^{(m-m)} = x^0$$

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Remember: exponents are only defined for positive integers, so far.

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In order to make this Power Rule hold, we define: $x^0 = 1$

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Note: Because this came from division by x we need: $x \neq 0$

Power Rules of Exponents

In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$

Note: Our definition only makes sense if m is a positive integer

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If the power rule holds:

$$1 = \frac{x^m}{x^m} = x^{(m-m)} = x^0$$

Remember: exponents are only defined for positive integers, so far.

In order to make this Power Rule hold, we define: $x^0 = 1$

Note: Because this came from division by x we need: $x \neq 0$

Now we have defined exponents for positive integers and 0.

Power Rules of Exponents

Power Rules of Exponents

In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$ and $\underbrace{b^0 = 1}_{b \neq 0}$

Note: Our definition only makes sense if m is a positive integer or 0

$$\frac{x^m}{x^n} = \frac{\overbrace{x \cdots x}^{n-times} \quad \overbrace{x \cdots x}^{(m-n)-times}}{\underbrace{x \cdots x}_{n-times}} = x^{(m-n)}$$

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Let's look at when $m < n$

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To get the clearest picture, let's look when $m = 0$

$$\frac{x^0}{x^n} =$$

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$$\frac{1}{x^n} = \frac{x^0}{x^n} =$$

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If the power rule holds:

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Recall: exponents are only defined for positive integers and 0, so far.

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If the power rule holds:

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Recall: exponents are only defined for positive integers and 0, so far.
In order to make this Power Rule hold, we define: $x^{-n} = \frac{1}{x^n}$
So we now have exponents defined for *ALL* integers.

Power Rules of Exponents

In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$ and $\underbrace{b^0 = 1}_{b \neq 0}$

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Notice: We followed the same order of defining exponents as we did "inventing" numbers:

Power Rules of Exponents

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Notice: We followed the same order of defining exponents as we did "inventing" numbers:

First: Natural Numbers

Power Rules of Exponents

In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$ and $\underbrace{b^0 = 1}_{b \neq 0}$

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$$\frac{x^m}{x^n} = \frac{\overbrace{x \cdots x}^{n-times} \quad \overbrace{x \cdots x}^{(m-n)-times}}{\underbrace{x \cdots x}_{n-times}} = x^{(m-n)}$$

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Let's look at when $m < n$

To get the clearest picture, let's look when $m = 0$

If the power rule holds:

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Recall: exponents are only defined for positive integers and 0, so far.

In order to make this Power Rule hold, we define: $x^{-n} = \frac{1}{x^n}$

So we now have exponents defined for *ALL* integers.

Notice: We followed the same order of defining exponents as we did "inventing" numbers:

First: Natural Numbers; Second: 0

Power Rules of Exponents

In general: $b^m = \underbrace{a \cdot a \cdots a}_{b-times}$ and $\underbrace{b^0 = 1}_{b \neq 0}$

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First: Natural Numbers; Second: 0; Third: Negative Numbers

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So we now have exponents defined for *ALL* integers.

Notice: We followed the same order of defining exponents as we did "inventing" numbers:

First: Natural Numbers; Second: 0; Third: Negative Numbers

Following this progression, we will later include fractional exponents.