

## Exponentials in Banking - Compounded Continuously

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$$P(t) = 10000 \cdot \left(1 + \frac{6\%}{12}\right)^{12t}$$

Furthermore, we noticed that as we increased **the number of times per year that interest is compounded** then the **amount owed** increases.

Looking at different values of  $n$ :

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Because interest is compounded more and more (infinitely) often, we say that the interest is *compounded continuously*